CLASSICAL MECHANICS

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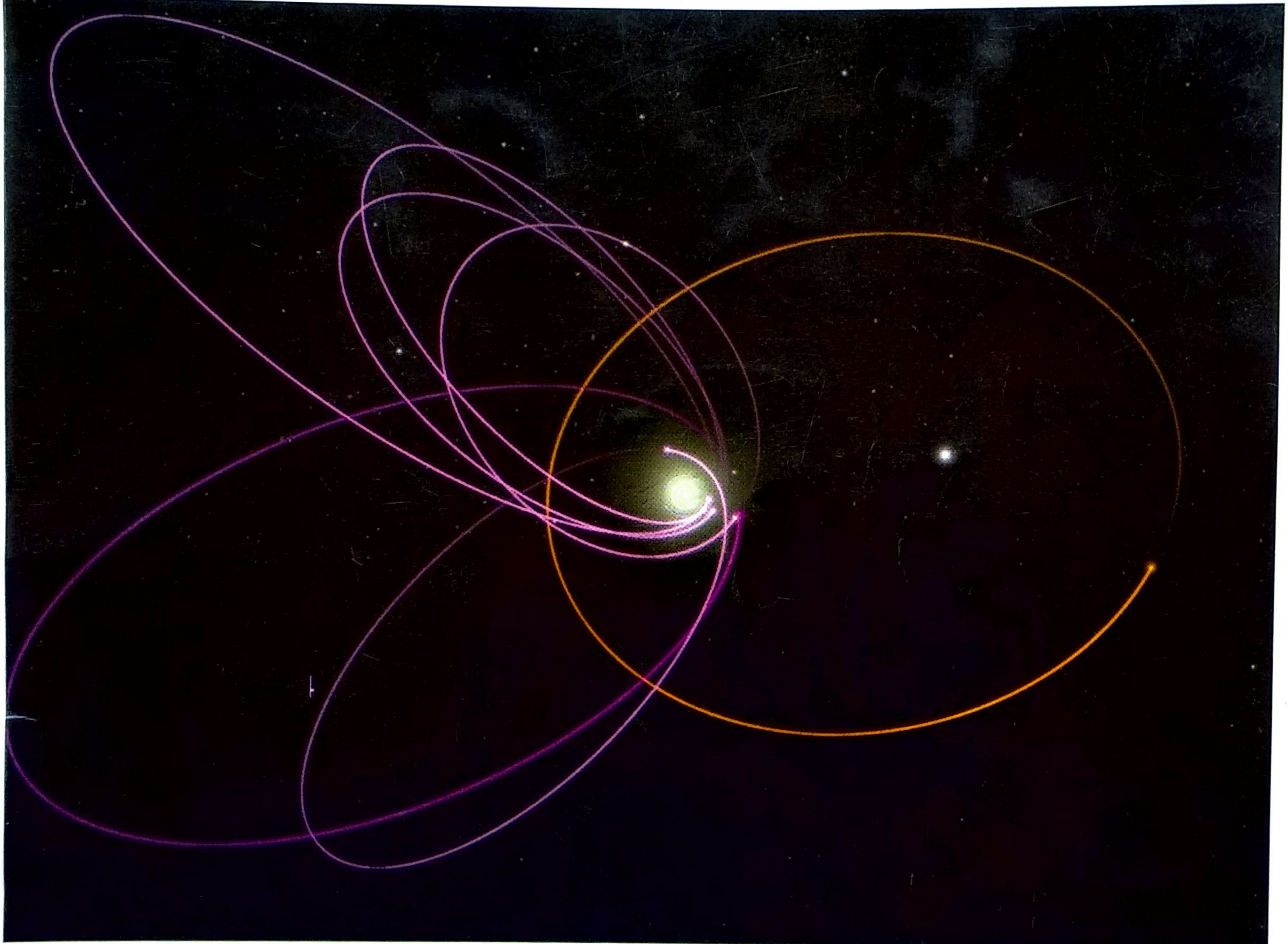

Assignment & Previous Years' Question

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LEVEL-1

Solve Yourself

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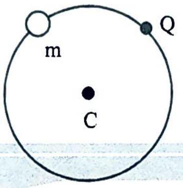
Assignment-1: Classical Mechanics (Review of Basic Concepts)

1. A ray of light is reflected from a mirror. If the mirror is slowly rotated with an angular velocity w, then the angle between the incident and the reflected rays changes at the rate
(a) –w (b) w (c) 2w (d) w/2

2. A point mass m is moving along a circle of radius r and centred at a point C. Q represents a fixed point on the circle (see figure). If w_C, w_Q be the instantaneous angular velocity of the mass around the points C and Q respectively, then



(a) $w_Q = 2w_C$ (b) $w_Q = w_C$ (c) $w_Q = \frac{w_C}{2}$ (d) w_Q and w_C are independent

3. A particle moves in two dimensions on the ellipse $x^2 + 4y^2 = 8$. At a particular instant it is at the point $(x, y) = (2, 1)$ and the x-component of its velocity is 6 (in suitable units). Then the y-component of its velocity is **[NET June 2015]**
(a) –3 (b) –2 (c) 1 (d) 4

4. If v_x, v_y be components of velocity in cartesian coordinate and v_r, v_θ be components of velocity in plane polar coordinate then transformation matrix for $(v_x, v_y) \to (v_r, v_\theta)$ is

(a) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ (b) $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ (c) $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ (d) $\begin{pmatrix} -\cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

5. A particle is moving along a circle $x^2 + y^2 = R^2$ with constant speed v_0. Average acceleration during the time it covers $\frac{1}{4}th$ of circle is

(a) zero (b) $\frac{2\sqrt{2}v_0^2}{R}$ (c) $\frac{\sqrt{2}v_0^2}{\pi R}$ (d) $\frac{2\sqrt{2}v_0^2}{\pi R}$

6. A particle of mass m moves with constant speed V_0 along a curve $y = \alpha x^2$. Force on the particle at point (0,0) is

(a) $m\alpha v_0^2$ (b) $2m\alpha v_0^2$ (c) $\frac{m\alpha v_0^2}{2}$ (d) $\frac{m\alpha v_0^2}{4}$

7. Polar coordinates of a particle vary with time as $r = Ae^{\omega t}, \theta = \omega t$, radial acceleration of the particle is :

(a) zero (b) $\omega^2 r$ (c) $2\omega^2 r$ (d) $\omega^2 r$

8. The value of quantity $\dfrac{x\ddot{x}+y\ddot{y}}{\sqrt{x^2+y^2}}$ in plane polar coordinate is

(a) $2\dot{r}\dot{\theta}+r\ddot{\theta}$ (b) $\ddot{r}-r\dot{\theta}^2$ (c) $\ddot{r}$ (d) $r\ddot{\theta}$

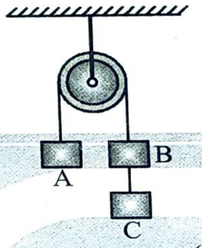
9. A particle is moving along the curve $y=ax-bx^2$. The point at which velocity is parallel to x-axis is

(a) $\dfrac{a}{2b},\dfrac{a^2}{2b}$ (b) $\dfrac{a}{2b},\dfrac{a^2}{3b}$ (c) $\dfrac{a}{4b},\dfrac{a^2}{4b}$ (d) $\dfrac{a}{2b},\dfrac{a^2}{4b}$

10. A ball weighing 100 gm, released from a height of 5 m, bounces perfectly elastically off a plate. The collision time between the ball and the plate is 0.5 s. The average force on the plate is approximately

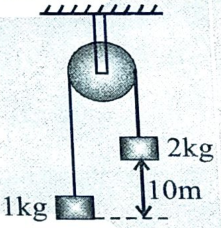
(a) 3 N (b) 2 N (c) 5 N (d) 4 N **[NET June 2017]**

11. Three equal weights A, B, C of mass 2kg each are hanging on a string passing over a fixed frictionless pulley as shown in the figure. The tension in the string connecting weights B and C is



(a) zero (b) 13 newton (c) 3.3 newton (d) 19.6 newton

12. Two blocks are held as shown in figure. On being released, how much time it takes for the two blocks t cross each other. (take g = 10 m/s^2)



(a) $\sqrt{3}$ sec

(b) $\sqrt{2}$ sec

(c) $\sqrt{6}$ sec

(d) 2sec

13. A block of mass 2 kg is at rest on a horizontal table. The coefficient of friction between the block an the table is 0.1. A horizontal force 3 N is applied to the block. The speed of the block (in m/s) after has moved a distance 10 m is____________________

14. A body of mass 10kg is being acted upon by a force 3 t^2 and an opposing constant force of 32 newt The initial speed is 10 m/s. The velocity of the body after 5 second is

(a) 6.5 m/s (b) 4.5 m/s (c) 3.5 m/s (d) 2.5 m/s

15. A ball of mass m is dropped from a tall building with zero initial velocity. In addition to gravity, the experiences a damping force of the form $-\gamma\upsilon$, where υ is its instantaneous velocity and γ is a const Given the values $m=10$ kg, $\gamma=10$ kg/s, and $g\approx 10$ m/s^2, the distance travelled (in metres) in time seconds, is **[NET Dec. 20**

(a) $10(t+1-e^{-t})$ (b) $10(t-1+e^{-t})$ (c) $5t^2-(1-e^{t})$ (d) $5t^2$

16. A small raindrop of mass m experiences a viscous drag force F_d = bv, proportional to its instantaneous s 'v'. If it starts from rest at a height h, its speed after a time 't' is

(a) $\upsilon(t)=\dfrac{mg}{b}\tanh\left(\dfrac{bt}{m}\right)$ (b) $\upsilon(t)=\dfrac{mg}{b}e^{-bt/m}$

(c) $\upsilon(t)=\dfrac{mg}{2b}(1-e^{-2bt/m})$ (d) $\upsilon(t)=\dfrac{mg}{b}(1-e^{-bt/m})$

GnhRV

[illegible] is not given viscous force.

17. Two solid spheres S_1 and S_2 of the same uniform density fall from rest under gravity in a viscous medium and, after some time, each terminal velocities, v_1 and v_2 respectively. If the masses of S_1 and S_2 are m_1 and m_2 respectively, and $v_1 = 4\,v_2$, then the ratio m_1/m_2 is **[TIFR 2011]**

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) 4 (d) 8

18. A constant force F is applied on a particle of mass '*m*' which is initially at rest. As the particle starts moving a resistive force $-bv$ begins to act on it. Speed of the particle at any instant of time '*t*' is

(a) $\frac{Ft}{m}$ (b) $\frac{F}{b}(1-e^{-\frac{b}{m}t})$ (c) $\frac{F}{m}t(1-e^{-\frac{b}{m}t})$ (d) $\frac{Ft}{m}t(1+e^{-\frac{bt}{m}})$

19. A particle of mass *m* is thrown with initial speed v_0. A resistance force $-Kv$ acts on the particle. Distance moved by the particle in time *t* is

(a) $\frac{mv_0}{K}$ (b) v_0t (c) $\frac{mv_0}{K}(1-e^{-\frac{K}{m}t})$ (d) $\frac{mv_0}{2K}(1-e^{-\frac{2K}{m}t})$

20. A body of mass *m* kg is rotating in a vertical circle at the end of a string of length *r* metre. The difference in the kinetic energy at the top and the bottom of the circle is

(a) $6mgr$ (b) $4mgr$ (c) $2mgr$ (d) mgr

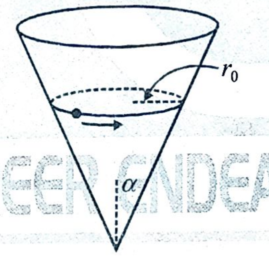
21. A particle is moving in a circle with uniform speed. It has constant

(a) velocity (b) acceleration (c) kinetic energy (d) displacement

22. A particle of mass *m* is rotating by means of a string in a vertical circle. The difference in the tensions at the bottom and the top would be

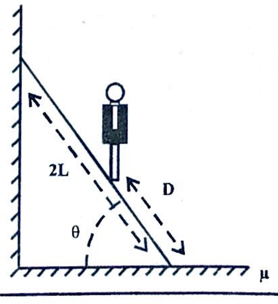
(a) 6 *mg* (b) 4 *mg* (c) 2 *mg* (d) zero

23. A particle slides on the inside surface of a frictionless cone. The cone is fixed with its tip on the ground and its axis vertical, as shown in the figure on the right. The semi-vertex angle of the cone is α. If the particle moves in a circle of radius r_0, without slipping downwards, the angular frequency w of this motion will be **[TIFR 2015]**



(a) $\sqrt{\frac{g}{r_0\cos\alpha}}$ (b) $\sqrt{\frac{g}{r_0\sin\alpha}}$ (c) $\sqrt{\frac{g}{r_0\cot\alpha}}$ (d) $\sqrt{\frac{g}{r_0\tan\alpha}}$

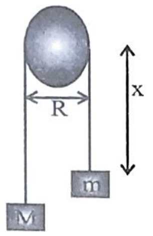
24. A uniform ladder of length 2L and mass 'm' leans against a wall in a vertical plane at an angle θ to the horizontal. The floor is rough, having a coefficient of static friction μ. A person of mass M stands on the ladder at a distance D from its base (see figure). If the wall is frictionless, the maximum distance (D_{max}) up the ladder that the person can reach before the ladder slips is **[TIFR 2014]**



(a) $2\mu L\left(1+\dfrac{m}{M}\right)\tan\theta$ (b) $\left\{2\mu\left(1+\dfrac{m}{M}\right)\tan\theta-\dfrac{m}{M}\right\}L$

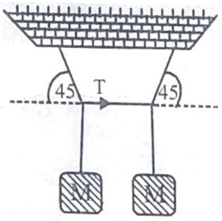
(c) $\mu L\tan\theta$ (d) $2\mu L\dfrac{m}{M}\tan\theta$

25. A heavy uniform rope of length L and mass per unit length μ goes over a frictionless pulley of diameters R, and has two masses M and m attached to its two ends as shown. In terms of the distance x, the equilibrium position is given by



(a) $\frac{1}{2}\left(L-\frac{\mu L}{m+M}+\frac{\pi R}{2}\right)$ (b) $\frac{1}{2}\left(L+\frac{m+M}{\mu}\right)$ (c) $L-\frac{\pi R}{2}$ (d) $\frac{1}{2}\left(L-\frac{m-M}{\mu}-\frac{\pi R}{2}\right)$

26. If the mechanical system shown below is in static equilibrium then what is then value of tension T ?

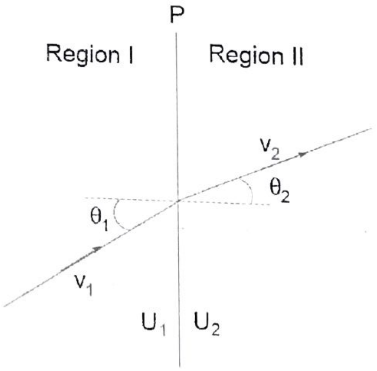


(a) mg (b) $\sqrt{2}$ mg (c) mg / $\sqrt{2}$ (d) 2mg

27. A ball is dropped vertically from a height H on to a plane surface and permitted to bounce repeatedly along a vertical line. After every bounce, its kinetic energy becomes a quarter of its kinetic energy before the bounce. The ball will come to rest after time **[TIFR 2016]**

(a) infinity (b) $(2H/g)^{1/2}$ (c) $2(2H/g)^{1/2}$ (d) $3(2H/g)^{1/2}$

28. A region of space is divided into two parts by a plane P, as shown in the figure below. A particle of mass m passes from Region I to Region II, where it has speed v_1 and v_2 respectively. There is a constant potential U_1 in Region I and U_2 in Region II. **[TIFR 2011]**



Let T_1 be the kinetic energy of the particles in Region I. If the trajectory of the particle is inclined to the normal to the plane P by angles θ_1 and θ_2, as shown in the figure then the ratio $\sin\theta_1/\sin\theta_2$ is given by

(a) $\sqrt{1-T_1/(U_1-U_2)}$ (b) $\sqrt{1+T_1/(U_1-U_2)}$ (c) $\sqrt{1-(U_1+U_2)/T_1}$

(d) $\sqrt{1+(U_1+U_2)/T_1}$ (e) $\sqrt{1-T_1/(U_1-U_2)}$ (f) $\sqrt{1+(U_1-U_2)/T_1}$

29. Four forces are given below in Cartesian and spherical polar coordinates. **[GATE 2015]**

(i) $\vec{F}_1 = K\exp\left(\frac{-r^2}{R^2}\right)\hat{r}$ (ii) $\vec{F}_2 = K\left(x^3\hat{y} - y^3\hat{z}\right)$

(iii) $\vec{F}_3 = K\left(x^3\hat{x} + y^3\hat{y}\right)$ (iv) $\vec{F}_4 = K\left(\frac{\hat{\phi}}{r}\right)$

where K is a constant. Identify the correct option.

(a) (iii) and (iv) are conservative but (i) and (ii) are not

(b) (i) and (ii) are conservative but (iii) and (iv) are not

(c) (ii) and (iii) are conservative but (i) and (iv) are not

(d) (i) and (iii) are conservative but (ii) and (iv) are not

30. A particle of unit mass moves in the xy-plane in such a way that $\dot{x}(t) = y(t)$ and $\dot{y}(t) = x(t)$. We can conclude that it is in a conservative force-field which can be derived from the potential

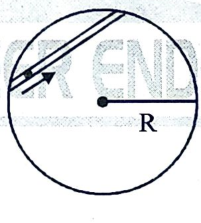
(a) $\frac{1}{2}(x^2 + y^2)$ (b) $\frac{1}{2}(x^2 - y^2)$ (c) $x + y$ (d) $x - y$ **[NET June 2015]**

31. The total mass of a spherical star of radius R and uniform density is M. If it explodes into a gas cloud of much lower density, the gravitational potential energy will

(a) decrease by $\frac{GM^2}{5R}$ (b) increase by $\frac{2GM^2}{5R}$

(c) increase by $\frac{3GM^2}{5R}$ (d) increase by $\frac{4GM^2}{5R}$ (e) decrease by $\frac{GM^2}{R}$

32. Consider the earth as a uniform (density) sphere of total mass 'M' and radius R. A small object slides along a tunnel connecting two points on the surface of the earth and is acted upon only by the gravitational force due to the earth. Using the value of the acceleration due to gravity as 9.81 metre/sec^2 and the radius of the earth R = 6.37×10^6 m, the time to slide between the two points is



(a) 21.1 min (b) 84.4 min (c) 42.2 min (d) 63.3 sec

33. In an experiment it is found that the combination $\frac{F}{\rho v^2}$ increases with a spatial dimension R as R^α. Here F is a force, ρ is a mass density and v is a velocity. The value of α is

(a) 4 (b) 1 (c) 3 (d) 2

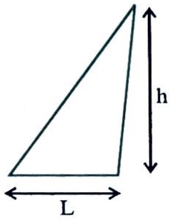
34. Raindrops hit the ground with a terminal velocity that is achieved due to a balance between the gravitational force and viscous drag force of air. Assume a raindrop to be a perfectly spherical water droplet of radius R. The momentum that it transfer when it hits the ground is proportional to

(a) R (b) R^2 (c) R^3 (d) R^5.

35. A high-velocity missile, travelling in a horizontal line with a kinetic energy of 3.0 Giga-Joules (GJ), explodes in flight and breaks into two pieces. A and B of equal mass. One of these pieces (A) flies off in a straight line perpendicular to the original direction in which the missile was moving and its kinetic energy is found to be 2.0 GJ. If gravity can be neglected for such high-velocity projectiles, it follows that the other piece (B) flew off in a direction at an angle with the original direction of **[TIFR 2012]**

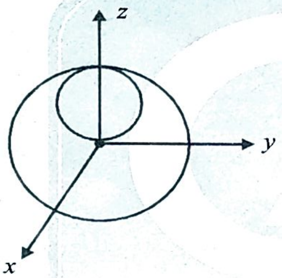
(a) 30° (b) $33^\circ 24'$ (c) 45° (d) 60°

36. A triangle of uniform mass density of base L and height h is shown below. The centre of mass of the triangle lies at this distance above the base:



(a) $\frac{h}{2}$ (b) $\frac{2h}{2}$ (c) $\frac{h}{3}$ (d) $\frac{h}{6}$

37. A spherical hole of diameter R has been scooped out of a solid sphere of uniform mass density and radius R. The hole is centred at $(x, y, z) = (0, 0, R/2)$ as shown in the figure



The centre of mass of the body is at

(a) $\left(0, 0, -\frac{R}{14}\right)$ (b) $\left(0, 0, \frac{R}{2}\right)$

(c) $\left(0, 0, \frac{R}{12}\right)$ (d) $\left(0, 0, -\frac{R}{8}\right)$ (e) $\left(0, 0, -\frac{R}{16}\right)$

38. Two particles each of mass m are fixed at $y = \pm a$. A particle of mass m is released from $x = -2\sqrt{2}a$, its speed when it reaches $x = 0$ due to gravitational attraction of fixed masses is

(a) $\sqrt{\frac{Gm}{a}}$ (b) $\sqrt{\frac{Gm}{3a}}$ (c) $\sqrt{\frac{2Gm}{3a}}$ (d) $\sqrt{\frac{8Gm}{3a}}$

39. A thin rod of length l has density $\rho = \rho_0 x^2 / l^2$, where x is distance from one end. The rod is suspended by two strings tied at its ends. If the strings are vertical and rod remains horizontal, ratio of tension in two strings is close to integer value ______________ (greater than one).

40. A bead of mass M slides along a parabolic wire described by $z = 2(x^2 + y^2)$. The wire rotates with angular velocity Ω about the z-axis. At what value of Ω does the bead maintain a constant nonzero height under the action of gravity along $-\hat{z}$? **[JEST 2017]**

(a) $\sqrt{3g}$ (b) $\sqrt{g}$ (c) $\sqrt{2g}$ (d) $\sqrt{4g}$

41. A particle of mass m falls from height h ($\ll$ radius of earth). A viscous drag force bv also acts on particle in addition to gravity where 'b' is constant and v is instantaneous speed. If g is acceleration due to gravity then acceleration of particle after time t is

(a) g (b) $ge^{-\frac{bt}{m}}$ (c) $ge^{\frac{bt}{m}}$ (d) $g\left(1+\frac{bt}{m}\right)$

42. A particle of unit mass is released at $x=1$ in one dimensional potential $V(x)=x^3-x$. Which one of the following statements is not correct

(a) Particle will come to rest at $x=0$ (b) Particle's speed is maximum at $x=\frac{1}{\sqrt{3}}$

(c) Speed of the particle at $x=\frac{1}{2}$ is $\frac{\sqrt{3}}{2}$ (d) Particle will not return to $x=1$

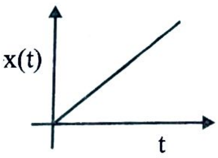
43. A particle of unit mass undergoes small oscillations about stable equilibrium point of one-dimensional potential $V(x)=x^2+\frac{1}{x^2}$. Angular frequency is

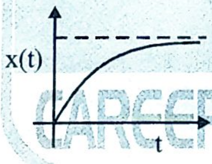
(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) 2 (d) $4\sqrt{2}$

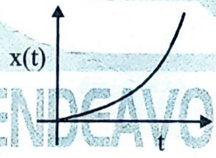
44. A simple pendulum, consisting of a small ball of mass 'm', attached to a massless string hanging vertically from the ceilling, is oscillating with an amplitude such that the maximum tension in the string is related to the minimum tension by $T_{max} = 2T_{min.}$ What is the value of the maximum tension is the string?

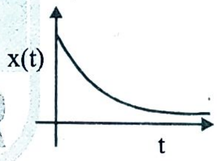
(a) mg/2 (b) 3mg/2 (c) 3mg/4 (d) 3mg

45. A particle is observed after it has been moving for a long time under the influence of a constant force in a medium that applies a drag force proportional to the square of its velocity. Distance versus time graph made by the observer will look like.

(a)


(b)


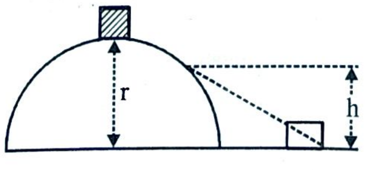
(c)


(d)


46. An object of mass m is thrown vertically up. It is acted upon by a constant resistive force F. If t_1 and t_2 be time of ascent and time of descent then value of t_1/t_2 is

(a) $\sqrt{\frac{g+F/m}{g-F/m}}$ (b) $\frac{g+F/m}{g-F/m}$ (c) $\frac{g-F/m}{g+F/m}$ (d) $\sqrt{\frac{g-F/m}{g+F/m}}$

47. A small body of mass m slides down from the top of a hemisphere of radius r. The surface of block and hemisphere are frictionless. The height at which the body lose contact with the surface of the sphere is

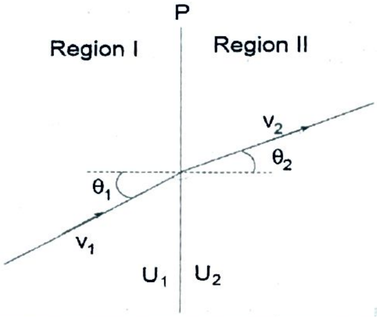


(a) $(3/2)r$ (b) $(2/3)r$ (c) $(1/2)g\,t^2$ (d) $v^2/2g$

48. A bob of mass m is attached to one end of a string of length l. This is kept in the horizontal position and released from rest. At what angle from vertical will the tension in the string be equal to the weight in magnitude?

(a) $\cos^{-1}\left(\frac{1}{2}\right)$ (b) $\cos^{-1}(1/3)$ (c) $\cos^{-1}(1/\sqrt{2})$ (d) $\cos^{-1}(1/\sqrt{3})$

49. A region of space is divided into two parts by a plane P, as shown in the figure below. A particle of mass m passes from Region I to Region II, where it has speed v_1 and v_2 respectively. There is a constant potential U_1 in Region I and U_2 in Region II. **[TIFR 2011]**

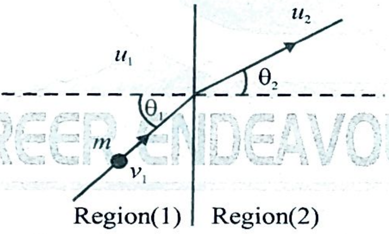


Let T_1 be the kinetic energy of the particles in Region I. If the trajectory of the particle is inclined to the normal to the plane P by angles θ_1 and θ_2, as shown in the figure then the ratio $\sin\theta_1/\sin\theta_2$ is given by

(a) $\sqrt{1-T_1/(U_1-U_2)}$ (b) $\sqrt{1+T_1/(U_1-U_2)}$ (c) $\sqrt{1-(U_1+U_2)/T_1}$ (d) $\sqrt{1+(U_1+U_2)/T_1}$

(e) $\sqrt{1-T_1/(U_1-U_2)}$ (f) $\sqrt{1+(U_1-U_2)/T_1}$

50. In the figure shown particle goes from region (1) to region (2). In region (1) it has speed v_1 and potential energy u_1 while in region (2) it has potential energy u_2. Which of the following is correct.



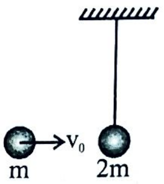
(a) $\frac{\sin\theta_1}{\sin\theta_2}=\left[1-\frac{2}{mv_1^2}(u_1-u_2)\right]^{\frac{1}{2}}$ (b) $\frac{\cos\theta_1}{\cos\theta_2}=\left[1+\frac{2}{mv_1^2}(u_1-u_2)\right]^{\frac{1}{2}}$

(c) $\frac{\sin\theta_1}{\sin\theta_2}=\left[1+\frac{2}{mv_1^2}(u_1-u_2)\right]^{\frac{1}{2}}$ (d) $\frac{\cos\theta_1}{\cos\theta_2}=\left[1-\frac{2}{mv_1^2}(u_1-u_2)\right]^{\frac{1}{2}}$

51. A particle of mass m uniformly accelerates from rest to speed v in time τ. Work done on the body in time t is

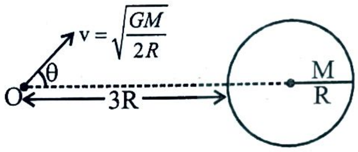
(a) $\frac{mv^2t^2}{\tau^2}$ (b) $\frac{mv^2t^2}{2\tau^2}$ (c) $\frac{2mv^2t^2}{\tau^2}$ (d) $\frac{mv^2t^2}{\tau^2}$

52. In figure shown if moving particle sticks to the suspended particle and $v_0 = 3\sqrt{gl}$ then the angle by which string will deflect is



(a) 30° (b) 45° (c) $\cos^{-1}\frac{1}{3}$ (d) 60°

53. In the figure shown a particle is thrown from point 'O' towards a heavy sphere. If particle is only acted upon by gravitational force of sphere then for what minimum value of angle θ it will not hit the sphere.

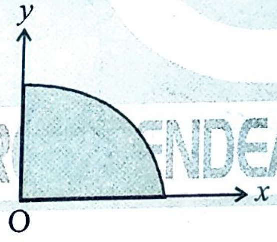


(a) 45° (b) 60° (c) $\sin^{-1}\frac{2}{3}$ (d) 30°

54. A non-uniform rod of length L is lying along positive x-axis with one end at origin. The rod has mass per unit length, which varies with x as $\lambda = \frac{Kx^2}{L}$, where K is a constant and x is the distance from one end at origin. The x co-ordinates of centre of mass is

(a) $\frac{3L}{4}$ (b) $\frac{L}{8}$ (c) $\frac{K}{L}$ (d) $\frac{3K}{L}$

55. Figure shows a quarter of circular disc of radius R. Coordinate of centre of mass of the disc is



(a) $\frac{4\pi}{3\pi}, \frac{4R}{3\pi}$ (b) $\frac{2\pi}{3\pi}, \frac{2R}{3\pi}$ (c) $\frac{R}{3\pi}, \frac{R}{3\pi}$ (d) $\frac{2R}{\pi}, \frac{2R}{\pi}$

Answer Key

1. (c)	2. (c)	3. (a)	4. (a)	5. (d)	6. (b)	7. (a)
8. (b)	9. (d)	10. (d)	11. (b)	12. (c)	13. (3 to 4)	14. ()
15. (b)	16. (d)	17. (d)	18. (b)	19. (c)	20. (c)	21. ()
22. (a)	23. (d)	24. (b)	25. (d)	26. (a)	27. (d)	28. (f)
29. (d)	30. (a)	31. (c)	32. (c)	33. (d)	34. (d)	35. (a)
36. (c)	37. (a)	38.(d)	39. (3)	40. (d)	41. (b)	42. (d)
43. (b)	44. (b)	45. (a)	46. (d)	47. (b)	48. (b)	49. (f)
50. ( )	51. (b)	52. (d)	53. (d)	54. (a)	55. (a)	

CSIR-UGC-NET/JRF | GATE PHYSICS

Assignment-02: Classical Mechanics (Lagrangian Dynamics)

1. The dimension of action are

(a) ML^2T^{-2} (b) MLT^{-2} (c) MLT^{-1} (d) ML^2T^{-1}

2. Lagrangian for a charged particle with charge e and mass m moving in an electromagnetic field with scalar potential $\phi(x,y,z)$ and vector potential $\vec{A}(x,y,z)$ is given

$$\vec{L} = \frac{1}{2}mv^2 - e\phi + \frac{e}{c}\vec{A}\cdot\vec{v}$$

Expression for generalised momentum is

(a) zero (b) $m\vec{v}$ (c) $m\vec{v} + \frac{e}{c}\vec{A}$ (d) $m\vec{v} - e\phi + \frac{e}{c}\vec{A}$

3. The Lagrangian of a particle moving in a plane under the influence of a central potential is given by $\mathrm{L} = \frac{1}{2}\mathrm{m}\left(\dot{\mathrm{r}}^2 + \mathrm{r}^2\dot{\theta}^2\right) - \mathrm{V}(\mathrm{r})$. The generalized momenta corresponding to r and θ are given by:

(a) $\mathrm{m}\dot{\mathrm{r}}$ and $\mathrm{mr}^2\dot{\theta}$ (b) $m\dot{r}$ and $mr\,\dot{\theta}$ (c) $\mathrm{m}\dot{\mathrm{r}}^2$ and $\mathrm{mr}^2\dot{\theta}$ (d) $\mathrm{m}\dot{\mathrm{r}}^2$ and $\mathrm{mr}^2\dot{\theta}^2$

4. Generalised potential for a system is $U(q,\dot{q}) = \frac{(q+\dot{q})^2}{2}$, corresponding generalised force is

(a) $\dot{q} + \ddot{q}$ (b) $\ddot{q} - \dot{q}$ (c) $\ddot{q} + q$ (d) $\ddot{q} - q$

5. The equation of motion of a system described by the time-dependent Lagrangian $L = e^{\gamma t}\left[\frac{1}{2}m\dot{x}^2 - V(x)\right]$ is

(a) $m\ddot{x} + \gamma m\dot{x} + \frac{dV}{dx} = 0$ (b) $m\ddot{x} + \gamma m\dot{x} - \frac{dV}{dx} = 0$ **[NET Dec. 2014]**

(c) $m\ddot{x} - \gamma m\dot{x} + \frac{dV}{dx} = 0$ (d) $m\ddot{x} + \frac{dV}{dx} = 0$

6. Lagrangian of a system is $L = e^{\gamma t}\left(\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2\right)$ where γ, m and k are positive constants. Equation of motion of system is,

(a) $\ddot{x} - \gamma\dot{x} + \frac{k}{m}x = 0$ (b) $\ddot{x} + \gamma\dot{x} - \frac{k}{m}x = 0$ (c) $\ddot{x} + \gamma\dot{x} + \frac{k}{m}x = 0$ (d) $\ddot{x} - \gamma\dot{x} - \frac{k}{m}x = 0$

7. The Lagrangian for a one-dimensional harmonic oscillator is

(a) $\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$ (b) $\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$ (c) $m\dot{x} + kx$ (d) $\frac{1}{2}\left(mx^2 + kx^2\right)$

Statement for Linked Q. 8 and Q.9.

A particle of mass m is constrained to move in a vertical plane along a trajectory given by $x = A\cos\theta$, $y = A\sin\theta$, where A is a constant: **[GATE]**

8. The Lagrangian of the particle is:

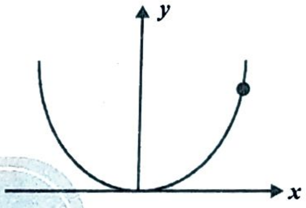
(a) $\frac{1}{2}mA^2\dot{\theta}^2 - mgA\cos\theta$ (b) $\frac{1}{2}mA^2\dot{\theta}^2 - mgA\sin\theta$

(c) $\frac{1}{2}mA^2\dot{\theta}^2$ (d) $\frac{1}{2}mA^2\dot{\theta}^2 + mgA\cos\theta$

9. The equation of motion of the particle is:

(a) $\ddot{\theta} + \frac{g}{A}\cos\theta = 0$ (b) $\ddot{\theta} + \frac{g}{A}\sin\theta = 0$ (c) $\ddot{\theta} = 0$ (d) $\ddot{\theta} - \frac{g}{A}\sin\theta = 0$

10. A bead of mass ‘m’ slides along a wire kept in vertical plane as shown in figure. The equation of the wire is $y = \alpha x^2$. Lagrangian of the bead is



(a) $\frac{1}{2}m\dot{x}^2\left(1+4\alpha^2x^2\right) - mg\alpha x^2$

(b) $\frac{1}{2}m\dot{x}^2\left(1+4\alpha^2x^2\right) + mg\alpha x^2$

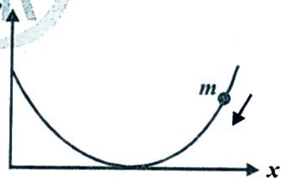
(c) $\frac{1}{2}m\dot{y}^2\left(1+\frac{1}{4y\alpha}\right) - mgy$ (d) $\frac{1}{2}m\dot{y}^2\left(1+\frac{1}{4y\alpha}\right) + mgy$

11. A projectile is thrown with intial speed ‘u’ at angle θ with horizontal. The projectile moves in x-y plane where ‘x’ axis is taken in horizontal and y-axis in vertical upward direction with origin being at point of projection. Lagrangian of projectile is:

(a) $\frac{1}{2}mu^2 - mgy$ (b) $\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right) - mgy$ (c) $\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right) + mgy$ (d) $\frac{1}{2}mu^2 - mgy\cos\theta$

12. A bead of mass ‘m’ slides in a cycloid shape wire kept in vertical plane as shown in figure. The parametric equation of the wire is $x = a(\theta - \sin\theta)$, $y = a(1+\cos\theta)$

Lagrangian of bead is



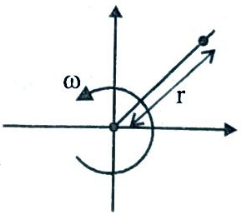
(a) $ma^2\dot{\theta}^2(1-\cos\theta) + mga$

(b) $ma^2\dot{\theta}^2(1+\cos\theta) - mga(1+\cos\theta)$

(c) $ma^2\dot{\theta}^2(1-\cos\theta) - mga(1+\cos\theta)$

(d) $ma^2\dot{\theta}^2 - mga(1+\cos\theta)$

13. A thin wire is rotated with uniform angular velocity ω in horizontal plane. A bead of mass ‘m’ slides on the wire. Distance of the bead from the fixed end of the wire as a function of time can be written as



(a) $r = a\cos\omega t + B\sin\omega t$

(b) $r = A\,e^{\omega t}$

(c) $r = A\,e^{-\omega t} + B\,e^{\omega t}$

(d) $r = A\tanh(\omega t)$

14. The parabolic coordinates (ξ, η) are related to the Cartesian coordinates (x, y) by $x = \xi\eta$ and $y = \frac{1}{2}(\xi^2 - \eta^2)$. The Lagrangian of a two-dimensional simple harmonic oscillator of mass m and angular frequency ω is

[NET Dec. 2016]

(a) $\frac{1}{2}m\left[\dot{\xi}^2 + \dot{\eta}^2 - \omega^2\left(\xi^2 + \eta^2\right)\right]$

(b) $\frac{1}{2}m\left(\xi^2 + \eta^2\right)\left[\left(\dot{\xi}^2 + \dot{\eta}^2\right) - \frac{1}{4}\omega^2\left(\xi^2 + \eta^2\right)\right]$

(c) $\frac{1}{2}m\left(\xi^2 + \eta^2\right)\left(\dot{\xi}^2 + \dot{\eta}^2 - \frac{1}{2}\omega^2\xi\eta\right)$

(d) $\frac{1}{2}m\left(\xi^2 + \eta^2\right)\left(\dot{\xi}^2 + \dot{\eta}^2 - \frac{1}{4}\omega^2\right)$

15. A particle of mass m moves along x-axis under the action of a force $F = \frac{-2k}{x^3}$. Its motion is described by the Lagrangian

(a) $\frac{m}{2}\left(\frac{dx}{dt}\right)^2 + \frac{k}{x^2}$ (b) $\frac{m}{2}\left(\frac{dx}{dt}\right)^2 + \frac{2k}{x^3}$ (c) $\frac{m}{2}\left(\frac{dx}{dt}\right)^2 - \frac{k}{x^2}$ (d) $\frac{m}{2}\left(\frac{dx}{dt}\right)^2 - \frac{2k}{x^3}$

16. The potential energy of a simple pendulum consisting of a bob of mass 'm' attached to a string of length 'l' displaced from the vertical by an angle 'θ' and allowed to oscillate (assume the potential energy to be zero at the rest position) will be

(a) $\frac{1}{2}m\ell^2\dot{\theta}^2 - mg\ell\cos\theta$

(b) $2mg\ell\sin^2\frac{\theta}{2}$

(c) $\frac{1}{2}m\ell\theta^2$

(d) $\frac{1}{2}g\ell\sin\theta$

17. With the positive z-axis chosen to be upwards, the Lagrangian of a particle of mass m falling under gravity is

(a) $\frac{1}{2}m\dot{z}^2 - mgz$ (b) $-\frac{1}{2}m\dot{z}^2 - mgz$ (c) $\frac{1}{2}m\dot{z}^2 + mgz$ (d) $-\frac{1}{2}m\dot{z}^2 + mgz$

18. Lagrangian of a system in cartesian coordinate is $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}k(\dot{x}y - \dot{y}x)$. Its Lagrangian in plane polar coordinates (r, θ) will be

(a) $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{1}{2}kr^2\dot{\theta}$

(b) $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{1}{2}kr^2\dot{\theta}$

(c) $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{1}{2}kr^2\dot{\theta}^2$

(d) $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{1}{2}kr^2\dot{\theta}^2$

19. The Lagrangian of a particle of mass m moving in a central potential V(r) is

$$L = \frac{1}{2}m\left(\dot{r}^2 + r\,\dot{\theta}^2 + r^2\sin^2\theta\dot{\varphi}^2\right) - V(r)$$

(a) θ is a cyclic coordinate

(b) θ and φ are cyclic coordinates

(c) φ is a cyclic coordinate

(d) r is a cyclic coordinate

20. If a generalised co-ordinate q is ignorable or cyclic in a Lagrangian L, then the following is conserved.

(a) $\frac{\partial L}{\partial q}$ (b) $\frac{\partial L}{\partial t}$ (c) $\frac{\partial L}{\partial \dot{q}}$ (d) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)$

21. Ignorable coordinates of the system corresponding to the Lagrangian
$L=\frac{1}{2}\left(\dot{\theta}^2+\dot{\phi}^2\sin^2\theta\right)+\frac{1}{2}\left(\dot{\psi}+\dot{\phi}\cos\theta\right)^2-k\cos\theta$, where θ,ϕ and ψ are generalized coordinates and k is a constant parameter, are

(a) θ and ϕ (b) ϕ and ψ (c) ψ and θ (d) θ,ϕ and ψ

22. The dynamics of a particle governed by the Lagrangian $L=\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2-kx\dot{x}t$ describes

(a) an undamped simple harmonic oscillator **[NET Dec. 2016]**
(b) a damped harmonic oscillator with a time varying damping factor
(c) an undamped harmonic oscillator with a time dependent frequency
(d) a free particle

23. The Lagrangian of a particle is given by $L=\dot{q}^2-q\dot{q}$. Which of the following statements is true ?

(a) This is a free particle **[JEST 2015]**
(b) The particle is experiencing velocity dependent damping
(c) The particle is executing simple harmonic motion
(d) The particle is under constant acceleration

24. The Lagrangian of a particle of mass m moving in one dimensions is given by **[NET June 2013]**

$$L=\frac{1}{2}m\dot{x}^2-bx$$

where b is a positive constant. The coordinate of the particle x(t) at time t is given by: (in the following c_1 and c_2 are constants)

(a) $-\frac{b}{2m}t^2+c_1t+c_2$ (b) c_1t+c_2

(c) $c_1\cos\left(\frac{bt}{m}\right)+c_2\sin\left(\frac{bt}{m}\right)$ (d) $c_1\cos h\left(\frac{bt}{m}\right)+c_2\sin h\left(\frac{bt}{m}\right)$

25. Lagrangian of a particle is $L=\frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2)-V(r)$. Which of the following options is correct ?

(a) $\dot{\theta}\propto\frac{1}{r}$ (b) $\dot{\theta}\propto\frac{1}{r^2}$ (c) $\dot{\theta}\propto r$ (d) $\dot{\theta}\propto r^2$

26. Lagrangian of a system is $L=\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2\cdot\dot{x}$ versus x graph is

(a) straight line (b) circle (c) ellipse (d) parabola

27. Lagrangian of a system is $L=\dot{q}_k\,q_k-\sqrt{1-\dot{q}_k^2}$, equation of motion of the system is

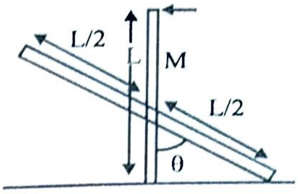
(a) $\ddot{q}_k+q_k=0$ (b) $\ddot{q}_k=0$ (c) $\ddot{q}_k-q_k=0$ (d) $\ddot{q}_k+q_k\dot{q}_k=0$

28. Lagrangian of a system is $L=a\dot{x}^2-bx^2$ then

(a) $x=c_1t+c_2t^2+c_3$ (b) $x=c_1e^{-c_2t}+c_3$

(c) $x=c_1\sin(c_2t+c_3)$ (d) $x=c_1\,e^{-c_2t}\sin(c_2t+c_3)$

29. A thin rod of mass 'M' and length 'L' is hold vertical on a smooth table. The rod is slightly displaced from its position due to which its centre falls vertically downward. If angle θ made with vertical is taken as generalised coordinate Lagrngian of the rod is



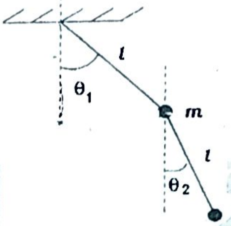
(a) $\frac{mL^2}{4}\left(\frac{1}{3}+\frac{1}{2}\sin^2\theta\right)\dot{\theta}^2-\frac{MgL\sin\theta}{2}$

(b) $\frac{ML^2}{8}\left(\frac{1}{3}+\sin^2\theta\right)\dot{\theta}^2-\frac{MgL\cos\theta}{2}$

(c) $\frac{ML^2}{4}\left(\frac{1}{3}+\frac{1}{2}\cos^2\theta\right)\dot{\theta}^2-\frac{MgL\cos\theta}{2}$

(d) $\frac{ML^2}{4}\left(\frac{1}{3}+\frac{1}{2}\cos^2\theta\right)\dot{\theta}^2-\frac{MgL\sin\theta}{2}$

30. A double pendulum consists of two point masses m attached by massless strings of length l as shown in the figure: **[NET Dec. 2011]**



The kinetic energy of the pendulum is:

(a) $\frac{1}{2}m\ell^2\left[\dot{\theta}_1^2+\dot{\theta}_2^2\right]$

(b) $\frac{1}{2}m\ell^2\left[2\dot{\theta}_1^2+\dot{\theta}_2^2+2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1-\theta_2)\right]$

(c) $\frac{1}{2}m\ell^2\left[\dot{\theta}_1^2+2\dot{\theta}_2^2+2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1-\theta_2)\right]$

(d) $\frac{1}{2}m\ell^2\left[2\dot{\theta}_1^2+\dot{\theta}_2^2+2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1+\theta_2)\right]$

31. The Lagrangian for a symmetric top is given by $L=\frac{1}{2}I_1\left(\dot{\theta}^2+\dot{\varphi}^2\sin^2\theta\right)+\frac{1}{2}I_3\left(\dot{\psi}+\dot{\varphi}\cos\theta\right)^2-Mgl\cos\theta$ where θ,φ,ψ are the Euler angles. Then the following are conserved quantities:

(a) P_φ and P_ψ
(b) P_θ and P_φ
(c) P_θ and P_ψ
(d) P_0, P_φ and P_ψ

32. If F_r be radial component of resultant forcce acting on a particle of mass 'm' then in spherical polar coordinate system Newton's law in radial direction is written as,

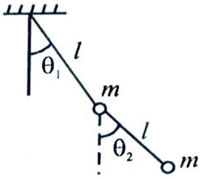
(a) $F_r = m\ddot{r}$

(b) $F_r = m\left(\ddot{r}-r\dot{\theta}^2\right)$

(c) $F_r = m\left(\ddot{r}-r\dot{\theta}^2-r\sin^2\theta\,\dot{\phi}^2\right)$

(d) $F_r = m\left(\ddot{r}-r\dot{\theta}^2-r\sin\theta\,\dot{\phi}^2\right)$

33. Equations of motion for small oscillation of double pendulum shown in figure are



(a) $2\ddot{\theta}_1+\ddot{\theta}_2+\frac{2g}{\ell}\theta_1=0,\ \ddot{\theta}_1+\ddot{\theta}_2+\frac{2g}{\ell}\theta_2=0$

(b) $2\ddot{\theta}_1+\ddot{\theta}_2+\frac{2g}{\ell}\theta_1=0,\ \ddot{\theta}_1+\ddot{\theta}_2+\frac{g}{\ell}\theta_2=0$

(c) $\ddot{\theta}_1+\ddot{\theta}_2+\frac{2g}{\ell}\theta_1=0,\ \ddot{\theta}_1+\ddot{\theta}_2+\frac{4g}{\ell}\theta_2=0$

(d) $\ddot{\theta}_1+2\ddot{\theta}_2+\frac{2g}{\ell}\theta_1=0,\ \ddot{\theta}_1+\ddot{\theta}_2+\frac{4g}{3\ell}\theta_2=0$

34. A bead of mass 'm' is constrained to move undergravity along a planar rigid wire that has parabolic shape $y=\frac{x^2}{\ell}$ where 'x' and 'y' are respectively horizontal and vertical coordinates. Lagrangian of the system is

(a) $\frac{1}{2}m\left(1+\frac{x^2}{\ell^2}\right)\dot{x}^2+\frac{mgx^2}{\ell}$ (b) $\frac{1}{2}m\left(1+\frac{2x^2}{\ell^2}\right)\dot{x}^2+\frac{mgx^2}{\ell}$

(c) $\frac{1}{2}m\left(1+\frac{x^2}{\ell^2}\right)\dot{x}^2-\frac{mgx^2}{\ell}$ (d) $\frac{1}{2}m\left(1+\frac{4x^2}{\ell^2}\right)\dot{x}^2-\frac{mgx^2}{\ell}$

35. If the Lagrangian $L_0=\frac{1}{2}m\left(\frac{dq}{dt}\right)^2-\frac{1}{2}m\omega^2q^2$ is modified to $L=L_0+aq\left(\frac{dq}{dt}\right)$, which one of the following is TRUE? **[GATE 2017]**

(a) Both the canonical momentum and equation of motion do not change
(b) Canonical momentum changes, equation of motion does not change
(c) Canonical momentum does not change, equation of motion changes
(d) Both the canonical momentum and equation of motion change

36. A possible Lagrangian for a free particle is **[JEST 2017]**

(a) $L=\dot{q}^2-q^2$ (b) $L=\dot{q}^2-q\dot{q}$ (c) $L=\dot{q}^2-q$ (d) $L=\dot{q}^2-\frac{1}{q}$

37. Lagrangian of a particle is $L=\frac{1}{2}m\dot{x}^2-\frac{1}{2}m\omega^2x^2-m\omega^2x\dot{x}t$. If at $t=0,\ x=0$ and $\dot{x}=v_0$ then velocity of particle at the moment particle is at $x=A$ is

(a) v_0 (b) ωA (c) zero (d) $\sqrt{v_0^2-\omega^2A^2}$

38. Lagrangian of a particle is $L=\frac{1}{2}m\dot{x}^2+bx\dot{x}$. Shape of phase space trajectory of the particle is

(a) circle (b) parabola (c) hyperbola (d) straight line

39. Lagrangian of a charged particle moving in electromagnetic potential $(\vec{A},\phi)$ is $L=\frac{1}{2}mv^2-q\phi+q\vec{A}\cdot\vec{v}$. If $(\vec{A},\phi)$ are transformed by using gauge transformation of electromagnetic potential $(\vec{A},\phi)\to(\vec{A}',\phi')$, $\vec{A}'=\vec{A}+\vec{\nabla}f(\vec{r},t)$, $\phi'=\phi-\frac{\partial f(r,t)}{\partial t}$. The transformed Lagrangian has canonical momentum.

(a) $m\vec{v}+q\vec{A}$ (b) $m\vec{v}$ (c) $m\vec{v}+q\vec{A}+q\vec{\nabla}f$ (d) $m\vec{v}+q\vec{A}-q\vec{\nabla}f$

40. Lagrangian of a particle is $L=\frac{1}{2}m(\dot{x}^2+\dot{y}^2)-\alpha(x\dot{y}-y\dot{x})$. If at $t=0, x=0, y=0, \dot{x}=v_0, \dot{y}=0$ then speed of the particle at $t=t$ will be

(a) v_0 (b) $v_0 \cos\frac{\alpha t}{m}$ (c) $v_0 e^{\left(\frac{\alpha t}{m}\right)}$ (d) $v_0 \cos^2\frac{\alpha t}{m}$

41. Lagrangian of a particle is $L=\frac{1}{2}m\dot{x}^2-\frac{1}{2}m\omega^2x^2$. If at $t=0$, $x=A$, $\dot{x}=0$, then at what time particle reaches $x=\frac{A}{2}$ for the first time.

(a) $\frac{\pi}{\omega}$ (b) $\frac{\pi}{2\omega}$ (c) $\frac{\pi}{3\omega}$ (d) $\frac{\pi}{4\omega}$

42. Lagrangian of a particle is $L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)-\frac{1}{2}m\omega^2\left(x^2+y^2\right)$. At $t=0, x=0, y=y_0$, $\dot{x}=\dot{y}=v_0$, angular momentum of the particle at time 't' is

(a) mv_0y_0 (b) $mv_0y_0\cos\omega t$ (c) $mv_0y_0\cos^2\omega t$ (d) $mv_0y_0e^{\omega t}$

43. A thin rod of length 'l' and mass 'm' is suspended from one end. The rod is deflected by 60° from vertical position and released. If the rod rotates freely about one end and motion remains confined to vertical plane, then which of the following statements is NOT correct.

(a) Energy is conserved but angular momentum is not conserved

(b) Lagrangian of the rod in terms of angle θ with downward vertical is $L=\frac{1}{6}m\ell^2\dot{\theta}^2+\frac{mg\ell}{2}\cos\theta$

(c) Angular velocity of the rod at the lowest point is $\sqrt{\frac{3g}{\ell}}$

(d) Angular acceleration of the rod at the initial moment is $-\frac{3\sqrt{3}g}{4\ell}$

Answer Key

1. (d)	2. (c)	3. (a)	4. (d)	5. (a)	6. (c)	7. (a)
8. (b)	9. (a)	10. (c)	11. (b)	12. (c)	13. (c)	14. (b)
15. (a)	16. (b)	17. (a)	18. (b)	19. (c)	20. (c)	21. (b)
22. ()	23. (a)	24. (a)	25. (b)	26. (c)	27. (b)	28. (c)
29. (b)	30. (c)	31. (a)	32. (c)	33. (b)	34. (d)	35. (b)
36. (b)	37. (a)	38. (d)	39. (c)	40. (a)	41. (c)	42. (a)
43. (c)						

CSIR-UGC-NET/JRF | GATE PHYSICS

Assignment-03: Classical Mechanics (Hamiltonian Dynamics)

1. If the Lagrangian of a particle moving in one dimensions is given by $L=\frac{\dot{x}^2}{2x}-V(x)$, the Hamiltonian is:

(a) $\frac{1}{2}xp^2+V(x)$ (b) $\frac{\dot{x}^2}{2x}+V(x)$ (c) $\frac{1}{2}\dot{x}^2+V(x)$ (d) $\frac{p^2}{2x}+V(x)$ **[NET June 2012]**

2. Lagrangian of a system is $e^{\gamma t}\left[\dot{x}^2-x^2\right]$, hamiltonian of the system is:

(a) $p_x^2\, e^{-\gamma t}+x^2\, e^{\gamma t}$ (b) $\frac{p_x^2}{2}e^{\gamma t}-\frac{x^2}{2}e^{-\gamma t}$ (c) $\frac{p_x^2}{4}e^{-\gamma t}+x^2\, e^{\gamma t}$ (d) $\frac{p_x^2}{2}e^{\gamma t}+\frac{x^2}{2}e^{-\gamma t}$

3. Lagrangian of a system is $L=\frac{1}{2}mv^2-\alpha\vec{r}.\vec{v}$ where $\vec{v}$ is velocity and α is a constant. Hamiltonian of the system is:

(a) $\frac{(\vec{p}-\alpha\vec{r})^2}{2m}$ (b) $\frac{(\vec{p}+\alpha\vec{r})^2}{2m}$ (c) $\frac{p^2}{2m}+\frac{\alpha\vec{p}.\vec{r}}{m}$ (d) $\frac{p^2}{2m}-\frac{\alpha\vec{p}.\vec{r}}{m}$

4. Lagrangian of a system is $L=\frac{1}{2}m(\vec{v}+\vec{a})^2$, hamiltonian of the system is ($\vec{a}$ is constant and $\vec{v}$ is velocity of particle)

(a) $\frac{p^2}{2m}+\vec{a}.\vec{p}$ (b) $\frac{p^2}{2m}-\vec{a}.\vec{p}$ (c) $\frac{(\vec{p}+m\vec{a})^2}{2m}$ (d) $\frac{(\vec{p}-m\vec{a})^2}{2m}$

5. Lagrangian of a system is $L=\frac{1}{2}m\dot{x}^2\left[1+\left(\frac{df}{dx}\right)^2\right]-mgf(x)$, its Hamiltonian is

(a) $\frac{p_x^2}{2m}\left[1+\left(\frac{df}{dx}\right)^2\right]+mg\, f(x)$ (b) $\frac{p_x^2}{2m}\left(\frac{df}{dx}\right)^2+mg\, f(x)$

(c) $\frac{p_x^2}{2m\left[1+\left(\frac{df}{dx}\right)^2\right]}+mg\, f(x)$ (d) $\frac{p_x^2}{2m\left(\frac{df}{dx}\right)^2}+mg\, f(x)$

6. Lagrangian of a system is $L=\frac{\dot{x}^2}{2}-\frac{1}{2}\omega^2x^2-\alpha x^3+\beta x\dot{x}^2$ hamiltonian of the system is

(a) $\frac{p_x^2}{2(1+2\beta x)^2}+\frac{1}{2}\omega^2+\alpha x^3$ (b) $\frac{p_x^2}{2(1+2\beta x)}+\frac{1}{2}\omega^2x^2+\alpha x^3$

(c) $\frac{p_x^2}{2(1+2\beta x)}-\frac{1}{2}\omega^2x^2-\alpha x^3$ (d) $\frac{2p_xx-p_x^2}{2(1+2\beta x)}-\frac{1}{2}\omega^2x^2+\alpha x^3$

7. If Lagrangian of a particle is $L=\dot{r}^2+br^2\dot{\theta}^2-kr\cos\theta$. Hamiltonian of the particle is

(a) $\frac{p_r^2}{4a}-\frac{p_\theta^2}{4br^2}+kr\cos\theta$ (b) $\frac{p_r^2}{4a}-\frac{p_\theta^2}{4br^2}-kr\cos\theta$

(c) $\frac{p_r^2}{4a}+\frac{p_\theta^2}{4br^2}+kr\cos\theta$ (d) $\frac{p_r^2}{4a}+\frac{p_\theta^2}{br^2}+kr\cos\theta$

8. Lagrangian of a particle is $L=a\dot{r}^2+b\dot{\theta}^2r^2+c\sin^2\theta\,\dot{\phi}^2r^2-V(r,\theta)$, where a,b,c are constant. Hamiltonian of the system is

(a) $\frac{p_r^2}{2a}+\frac{p_\theta^2}{2br^2}+\frac{p_\phi^2}{2cr^2\sin^2\theta}+V(r,\theta)$ (b) $\frac{a\,p_r^2}{2}+\frac{b\,p_\theta^2}{2r^2}+\frac{c\,p_\phi^2}{2r^2\sin^2\theta}+V(r,\theta)$

(c) $\frac{p_r^2}{4a}+\frac{p_\theta^2}{4br^2}+\frac{p_\phi^2}{4cr^2\sin^2\theta}+V(r,\theta)$ (d) $\frac{a\,p_r^2}{4}+\frac{b\,p_\theta^2}{4r^2}+\frac{c\,p_\phi^2}{4r^2\sin^2\theta}+V(r,\theta)$

9. The Hamiltonian of a system is independent of the coordinate q and is given by $\mathrm{H}=\mathrm{ap}^4$ where a is a constant. The corresponding Lagrangian is then given by

(a) $\mathrm{L}=\mathrm{m}\dot{\mathrm{q}}^2-\mathrm{ap}^4$ (b) $\mathrm{L}=\frac{1}{2}\mathrm{m}\dot{\mathrm{q}}^2-\mathrm{ap}^4$ (c) $\mathrm{L}=\mathrm{a}\dot{\mathrm{q}}^4$ (d) $\mathrm{L}=\frac{3\dot{\mathrm{q}}}{4}\left(\frac{\dot{\mathrm{q}}}{4\mathrm{a}}\right)^{1/3}$

10. The Hamiltonian of a relativistic particle of rest mass m and momentum p is given by $H=\sqrt{p^2+m^2}+V(x)$, in units in which the speed of light $c=1$. The corresponding Lagrangian is **[NET Dec. 2013]**

(a) $L=m\sqrt{1+\dot{x}^2}-V(x)$ (b) $L=-m\sqrt{1-\dot{x}^2}-V(x)$

(c) $L=\sqrt{1+m\dot{x}^2}-V(x)$ (d) $L=\frac{1}{2}m\dot{x}^2-V(x)$

11. Hamiltonian of a particle is $H=\frac{xp_x^2}{2}+x^2p_x+V(x)$ corresponding Lagrangian is

(a) $\frac{(\dot{x}-x^2)^2}{2x}-V(x)$ (b) $\frac{(\dot{x}+x^2)^2}{2x}+V(x)$

(c) $\frac{\dot{x}^2}{2x}-x^2\dot{x}-V(x)$ (d) $\frac{\dot{x}^2}{2x}+x^2\dot{x}-V(x)$

12. If Hamiltonian of a particle is $H=p_x^2+p_y^2-p_xp_y-\frac{1}{2}(x^2+y^2)$, then Lagrangian of the particle is

(a) $\frac{1}{3}(\dot{x}^2+\dot{y}^2+\dot{x}\dot{y})+\frac{1}{2}(x^2+y^2)$ (b) $\frac{1}{3}(\dot{x}^2+\dot{y}^2-\dot{x}\dot{y})+\frac{1}{2}(x^2+y^2)$

(c) $\frac{1}{3}(\dot{x}^2-\dot{y}^2)+\frac{1}{2}(x^2+y^2)$ (d) $\dot{x}^2+\dot{y}^2+\dot{x}\dot{y}+\frac{1}{2}(x^2+y^2)$

13. If the Lagrangian of a system has only one term which has a linear dependence on velocity, then its Hamiltonian
(a) Also has only one term depending on velocity.
(b) Is zero.
(c) Is not defined.
(d) Is independent of velocity and is constant.

14. A Dynamical system with two degrees of freedom, has generalised coordinates q_1 and q_2, and kinetic energy

$$T = \lambda \dot{q}_1 \dot{q}_2$$

[TIFR 2012]

If the potential energy is $V(q_1, q_2) = 0$, the correct form of the Hamiltonian for this system is:

(a) $p_1 p_2/\lambda$ (b) $\lambda \dot{q}_1 \dot{q}_2$ (c) $(p_2\dot{q}_1 + p_2\dot{q}_2)/2$ (d) $(p_1 q_2 + p_2 q_1)/2$

15. A spring made of a given material has the restoring force $F = -kx^3$. The total energy of a point of mass m at the end
of this spring is :

[✓ (a)] $E = \frac{1}{2}m\dot{x}^2 + \frac{1}{4}kx^4$ (b) $E = \frac{1}{2}m\dot{x}^2 - \frac{1}{4}kx^4$ (c) $E = \frac{1}{2}m\dot{x}^2 + kx^3$ (d) $E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$

16. The Hamiltonian of a system as a function of the canonical conjugate cordinate q and momentum p is given by H(q,p). The Hamilton's canonical equations are given by

(a) $p = \frac{\partial H}{\partial q}, q = \frac{\partial H}{\partial p}$ (b) $p = -\frac{\partial H}{\partial \dot{q}}, q = \frac{\partial H}{\partial \dot{p}}$

(c) $\dot{p} = -\frac{\partial H}{\partial q}, \dot{q} = \frac{\partial H}{\partial p}$ (d) $\dot{p} = \frac{\partial H}{\partial q}, \dot{q} = -\frac{\partial H}{\partial q}$

17. Hamilton canonical equations of motion for a conservative system are:

(a) $-\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $-\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$ (b) $\frac{dp_i}{dt} = \frac{\partial H}{\partial p_i}$ and $\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$

(c) $-\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$ (d) $\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $-\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$

where q_i, p_i and H are generalized cooridnate, generalized momentum and Hamiltonian respectively.

18. If Lagrangian of a system $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$ then $\dot{p}_x$ is equal to

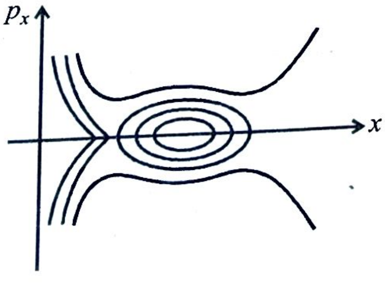
(a) $\frac{kp_x}{m}$ (b) kx (c) $-kx$ (d) $\frac{kx}{2}$

19. Hamiltonian of a particle is $H = \frac{p_x^2 + p_y^2}{2m} - \alpha x$. If at $t = 0$, $x = 0$, $y = 0$ and $\dot{x} = 0$, $\dot{y} = \beta$ the path of the particle on xy-plane will be

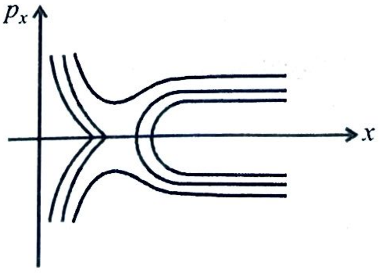
(a) straight line (b) parabola (c) circle (d) hyperbola

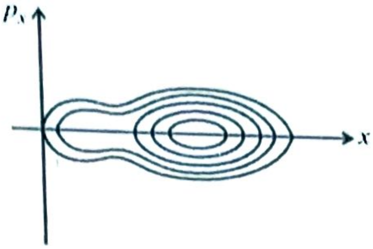
20. A particle is moving in potential $V(x) = \frac{1}{x^2} - \frac{1}{x^3}$. Its phase space trajectory will be

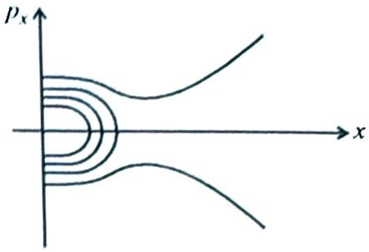
(a)



(b)



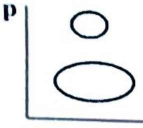
(c) 

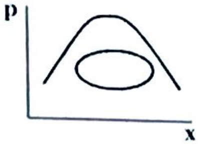
(d) 

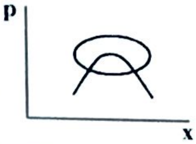
21. A particle is moving in potential $V(x) = -kx^4$ where $k > 0$. If total energy of the particle is zero then its phase space trajectory will be

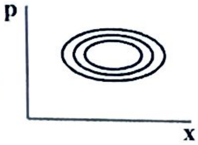
(a) straight line (b) parabola (c) hyperbola (d) ellipse

22. Which of the following set of phase-space trajectories which one is not possible for a particle obeying Hamilton's equations of motion (for a time-independent Hamiltonian)? **[NET Dec. 2012]**

(a) 

(b) 

(c) 

(d) 

23. The Hamiltonian for a system described by the generalized coordinate x and generalized momentum p is

$$H = \alpha x^2 p + \frac{p^2}{2(1+2\beta x)} + \frac{1}{2}\omega^2 x^2$$

where α, β and ω are constant. The corresponding Lagrangian is **[NET June 2017]**

(a) $\frac{1}{2}(\dot{x} - \alpha x^2)^2 (1+2\beta x) - \frac{1}{2}\omega^2 x^2$ (b) $\frac{1}{2(1+2\beta x)}\dot{x}^2 - \frac{1}{2}\omega^2 x^2 - \alpha x^2 \dot{x}$

(c) $\frac{1}{2}(\dot{x}^2 - \alpha^2 x)^2 (1+2\beta x) - \frac{1}{2}\omega^2 x^2$ (d) $\frac{1}{2(1+2\beta x)}\dot{x}^2 - \frac{1}{2}\omega^2 x^2 + \alpha x^2 \dot{x}$

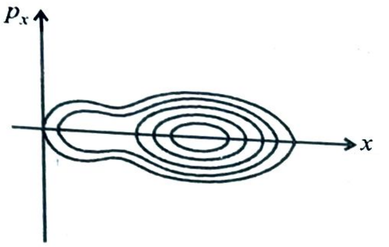
24. Lagrangian of a system is $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - a(x\dot{y} + y\dot{x})$. Hamiltonian of system will be

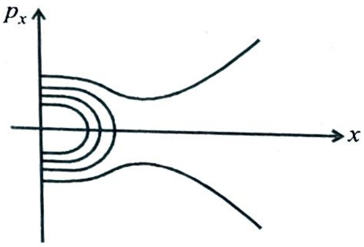
(a) $\frac{p_x^2 + p_y^2}{2m} + \frac{a^2(x^2+y^2)}{2m} + \frac{a}{m}(xp_y + yp_x)$

(b) $\frac{p_x^2 + p_y^2}{2m} - \frac{a^2(x^2+y^2)}{2m} + \frac{a}{m}(xp_y + yp_x)$

(c) $\frac{p_x^2 + p_y^2}{2m} + \frac{a^2(x^2+y^2)}{2m} + \frac{a}{m}(xp_y - yp_x)$

(d) $\frac{p_x^2 + p_y^2}{2m} + \frac{a^2(x^2+y^2)}{2m}$

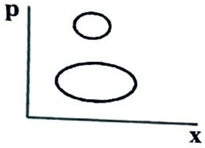
(c)


(d)


21. A particle is moving in potential $V(x)=-kx^4$ where $k>0$. If total energy of the particle is zero then its phase space trajectory will be

(a) straight line (b) parabola (c) hyperbola (d) ellipse

22. Which of the following set of phase-space trajectories which one is not possible for a particle obeying Hamilton's equations of motion (for a time-independent Hamiltonian)? **[NET Dec. 2012]**

(a)


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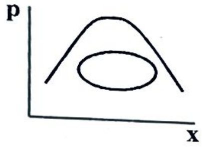

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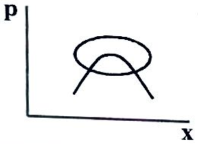
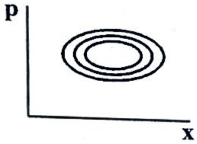


(d)


23. The Hamiltonian for a system described by the generalized coordinate x and generalized momentum p is

$$H=\alpha x^2 p+\frac{p^2}{2(1+2\beta x)}+\frac{1}{2}\omega^2 x^2$$

where α, β and ω are constant. The corresponding Lagrangian is **[NET June 2017]**

(a) $\frac{1}{2}(\dot{x}-\alpha x^2)^2(1+2\beta x)-\frac{1}{2}\omega^2 x^2$ (b) $\frac{1}{2(1+2\beta x)}\dot{x}^2-\frac{1}{2}\omega^2 x^2-\alpha x^2\dot{x}$

(c) $\frac{1}{2}(\dot{x}^2-\alpha^2 x)^2(1+2\beta x)-\frac{1}{2}\omega^2 x^2$ (d) $\frac{1}{2(1+2\beta x)}\dot{x}^2-\frac{1}{2}\omega^2 x^2+\alpha x^2\dot{x}$

24. Lagrangian of a system is $L=\frac{1}{2}m(\dot{x}^2+\dot{y}^2)-a(x\dot{y}+y\dot{x})$. Hamiltonian of system will be

(a) $\frac{p_x^2+p_y^2}{2m}+\frac{a^2(x^2+y^2)}{2m}+\frac{a}{m}(xp_y+yp_x)$

(b) $\frac{p_x^2+p_y^2}{2m}-\frac{a^2(x^2+y^2)}{2m}+\frac{a}{m}(xp_y+yp_x)$

(c) $\frac{p_x^2+p_y^2}{2m}+\frac{a^2(x^2+y^2)}{2m}+\frac{a}{m}(xp_y-yp_x)$

(d) $\frac{p_x^2+p_y^2}{2m}+\frac{a^2(x^2+y^2)}{2m}$

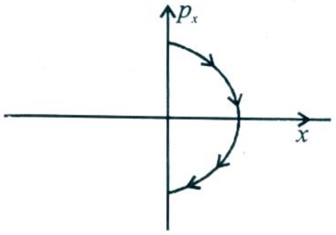
25. Hamiltonian of a system is $H=\frac{p_x^2}{2m}e^{-\gamma t}+\frac{1}{2}Kx^2e^{\gamma t}$ Lagrangian of a system is

(a) $e^{\gamma t}\left(\frac{1}{2}m\dot{x}^2-\frac{1}{2}Kx^2\right)$ (b) $e^{-\gamma t}\left(\frac{1}{2}m\dot{x}^2-\frac{1}{2}Kx^2\right)$

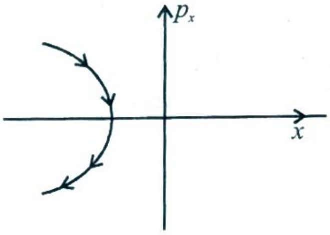
(c) $e^{-\gamma t}\frac{1}{2}m\dot{x}^2-e^{\gamma t}\frac{1}{2}Kx^2$ (d) $e^{\gamma t}\frac{1}{2}m\dot{x}^2-\frac{1}{2}Kx^2e^{-\gamma t}$

26. Phase space trajectory of a particle moving in potential $V(x)=x+2$ for energy $E=-2$ is

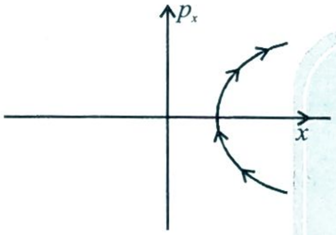
(a)



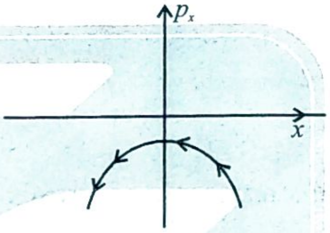
(b)



(c)



(d)



27. Hamiltonian of a system is $H=\sqrt{p_x^2+1}-x$. Shape of $x-t$ graph for the particle is

(a) straight line (b) parabola (c) hyperbola (d) ellipse

28. Hamiltonian of a particle moving on x-y plane is $H=\frac{(p_x+y)^2+(p_y-x)^2}{2m}$ which of the following is conserved.

(a) only H (b) H,p_x,p_y (c) H,L_z (d) H,p_x,p_y,L_z

29. A canonical transformation $(q,p\rightarrow Q,P)$ is made through the generating function $F(q,P)=q^2P$ on the Hamiltonian $H(q,p)=\frac{p^2}{4q^2}+q^4$. Shape of phase space trajectory on the Q – P plane is

(a) straight line (b) circle (c) parabola (d) hyperbola

30. Hamiltonian of a system is $H=\frac{p_x^2}{2m}+\frac{1}{2}Kx^2+\alpha x$. If phase space trajectory of the particle is circle then numerical value of angular frequency of small oscillation about stable equilibrium point is

(a) $\frac{1}{m}$ (b) $\frac{1}{2m}$ (c) $\frac{K}{m}$ (d) $2K$

31. For a dynamical system, time evolution of dynamical variable (x) is given as

$$\frac{dx}{dt} = x\sqrt{1-x^2}$$

Which one of the following statements is correct

(a) $x = 0$ is unstable fixed point and $x = \pm 1$ are stable fixed points

(b) $x = 0$ is stable fixed point and $x = \pm 1$ are unstable fixed points

(c) $x = 0, 1$ are stable fixed point and $x = -1$ is unstable fixed points

(d) $x = 0, -1$ are stable fixed point and $x = 1$ is unstable fixed points

32. Hamiltonian of a particle is $H = \frac{p_x^2}{2m} + kx^4$. If energy of the particle is doubled then area of the phase space trajectory will become

(a) double (b) half (c) $2^{1/2}$ times (d) $2^{3/4}$ times

33. For a dynamical system, time evolution of dynamical variable is given by

$$\frac{dx}{dt} = x - x^3$$

Which one of the following statement is correct

(a) There are two fixed points

(b) There is one stable and two unstable fixed points

(c) There is one unstable and two stable fixed points

(d) There are three stable fixed points

Answer Key

1. (a)	2. (c)	3. (b)	4. (b)	5. (c)	6. (b)	7. (c)
8. (c)	9. (d)	10. (b)	11. (a)	12. (b)	13. (a)	14. (a)
15. (c)	16. (d)	17. (c)	18. (b)	19. (b)	20. (b)	21. (b)
22. (c)	23. (a)	24. (a)	25. (a)	26. (b)	27. (c)	28. (c)
29. (b)	30. (a)	31. (b)	32. (d)	33. (c)		

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Assignment-04 : Classical Mechanics (Conservation Laws)

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1. A system exhibits invariance under translation in space and time. The following quantities will remain conserved.
(a) Linear momentum and angular momentum.
(b) Angular momentum and energy.
(c) Linear momentum and energy.
(d) Linear momentum, angular momentum and energy.

2. For a system described by the Lagrangian,

$$L = \frac{\mu}{2}\frac{d\vec{r}}{dt}\cdot\frac{d\vec{r}}{dt} + \lambda\left(x^2 + a^2y^2 + z^2\right) \qquad a \in \Re$$

which one of the following statements is correct.
(a) total energy and total angular momentum are conserved
(b) total energy and angular momentum about y-axis are conserved.
(c) total energy and linear momentum about y-axis are conserved.
(d) total energy and angular momentum about x-axis and z-axis are conserved.

3. One of the generalized co-ordinates Q_j of a system is cyclic. Then its
(a) Energy is not conserved.
(b) Conjugate momentum P_j is also cyclic.
(c) Conjugate momentum P_j is conserved.
(d) Conjugate momentum P_j is indeterminate.

4. Conservation of angular momentum is related to the invariance of Lagrangian under
(a) Rotation in space (b) Translation in space (c) Inversion in space (d) Translation in time

5. The dynamics of a system with generalized coordinates (x, y, z) is described by a Lagrangian $L = L(\dot{x}, \dot{y}, \dot{z}, x, y, t)$ where t is time. Which of the following statements is correct?
(a) The total energy and x-component of the linear momentum are conserved.
(b) The total energy is not conserved but y-component of the linear momentum is conserved.
(c) The total energy is conserved but z-component of linear momentum is not conserved.
(d) The total energy is not conserved but z-component of linear momentum is conserved.

6. The homogeneity of time leads to the law of conservation of:
(a) Linear momentum (b) Angular momentum (c) Energy (d) Parity

7. If $\frac{\partial L}{\partial q_n} = 0$, where L is the Lagrangian for a conservative system without constraints and q_n is a generalized coordinate, then the generalized momentum is:
(a) a cycle cooridnat
(b) a constant of motion
(c) equal to $\frac{d}{dt}\left(\frac{\partial L}{\partial q_n}\right)$
(d) undefined

8. The Lagrangian of a system with one degree of freedom q is given by $L = \alpha\dot{q}^2 + \beta q^2$, where α and β are non-zero constants. If p_q denotes the canonical momentum conjugate to q then which one of the following statements is CORRECT?

(a) $p_q = 2\beta q$ and it is a conserved quantity.
(b) $p_q = 2\beta q$ and it is not a conserved quantity.
(c) $p_q = 2\alpha\dot{q}$ and it is a conserved quantity.
(d) $p_q = 2\alpha\dot{q}$ and it is not a conserved quantity.

9. The Lagrangian of a system is given by

$L = \frac{1}{2}ml^2\left[\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2\right] - mgl\cos\theta$, where m, l and g are constants. **[GATE 2016]**

Which of the following is conserved?

(a) $\dot{\varphi}\sin^2\theta$
(b) $\dot{\varphi}\sin\theta$
(c) $\frac{\dot{\varphi}}{\sin\theta}$
(d) $\frac{\dot{\varphi}}{\sin^2\theta}$

10. The Lagrangian of a free particle in spherical polar co-ordinates is given by $L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\dot{\phi}^2\sin^2\theta\right)$. The quantity that is conserved is **[GATE 2009]**

(a) $\frac{\partial L}{\partial \dot{r}}$
(b) $\frac{\partial L}{\partial \dot{\theta}}$
(c) $\frac{\partial L}{\partial \dot{\phi}}$
(d) $\frac{\partial L}{\partial \dot{\phi}} + \dot{r}\dot{\theta}$

11. A particle in two dimensions is in a potential $V(x, y) = x + 2y$. Which of the following (apart from the total energy of the particle) is also a constant of motion? **[NET Dec. 2016]**

(a) $p_y - 2p_x$
(b) $p_x - 2p_y$
(c) $p_x + 2p_y$
(d) $p_y + 2p_x$

12. Lagrangian of a system is $e^{\gamma t}\left[\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2\right]$. If p_x and E represent the linear momentum and total energy then which of the following statement is correct.

(a) $p_x =$ constant
(b) $E =$ constant
(c) $p_x \neq$ constant, $E =$ constant
(d) $p_x \neq$ constant, $E \neq$ constant.

13. A particle is moving in a potential $\alpha x^2 + \beta y^2 + z^2$ for what values of α and β y-component of angular momentum is conserved.

(a) $\alpha = 2, \beta = 1$
(b) $\beta = 1$ and any value of α
(c) $\alpha = 1$ and any value of β
(d) $\alpha = 1$ and $\beta = 1$ only

14. A particle is moving in a potential $V(x, y, z) = \frac{k}{\sqrt{x^2 + y^2 + z^2}}$ which is correct.

(a) Only L_x is conserved
(b) Only L_y is conserved
(c) Only L_z is conserved
(d) All L_x, L_y, L_z are conserved.

15. A particle is moving under a potential $V(x, y) = k(x + y)$. Which of the following statement is correct.

(a) $p_x =$ constant
(b) $L_z =$ constant
(c) $p_x - p_y =$ constant
(d) $p_x + p_y =$ constant

Answer Key

1. (c) 2. (b) 3. (c) 4. (a) 5. (d) 6. (c) 7. (b)
8. (d) 9. (a) 10. (c) 11. (a) 12. (d) 13. (c) 14. (d)
15. (c)

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Assignment-05: Classical Mechanics (Small Oscillations)

1. In the free vibration of a linear triatomic molecule, the central atom takes part in the oscillatory motion for
(a) All three modes
(b) Two modes
(c) One mode only
(d) Does not take part at all

2. In a normal mode of vibration of a molecule, all atoms move
(a) With the same frequency
(b) With the same amplitude
(c) In the same direction
(d) Opposite to each other.

3. A linear symmetric triatomic molecule consisting of masses 1, 2 and 1 units connected by springs of unit force constant admits normal modes of following nonzero normal frequencies:
(a) 1, 1
(b) 1, 2
(c) $1, \sqrt{2}$
(d) $1, \sqrt{3}$

4. Consider two simple pendulums each consisting of a point mass m attached to a string of length ℓ. The masses are coupled by a spring of spring constant k. The kinetic energy T and the potential energy V of the system are given by $T = m\ell^2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $V = m\ell^2\begin{pmatrix} \frac{g}{\ell}+\frac{k}{m} & -\frac{k}{m} \\ -\frac{k}{m} & \frac{g}{\ell}+\frac{k}{m} \end{pmatrix}$
The frequencies of small oscillation of this system are
(a) $\omega_1 = \sqrt{g/\ell}, \omega_2 = \sqrt{g/\ell + 2k/m}$
(b) $\omega_1 = \sqrt{2k/m}, \omega_2 = \sqrt{g/\ell + 2k/m}$
(c) $\omega_1 = \sqrt{g/\ell}, \omega_2 = \sqrt{k/m}$
(d) $\omega_1 = \sqrt{g/\ell}, \omega_2 = \sqrt{g/\ell + k/m}$

Statement for Linked Answer Q.5 and Q.6:
The Lagrangian of two coupled oscillators of mass m each is:

$$L = \frac{1}{2}m\left(\dot{x}_1^2 + \dot{x}_2^2\right) - \frac{1}{2}m\omega_0^2\left(x_1^2 + x_2^2\right) + m\omega_0^2\mu x_1 x_2$$

5. The equations of motion are:
(a) $\ddot{x}_1 + \omega_0^2 x_1 = \omega_0^2\mu x_1, \ddot{x}_2 + \omega_0^2 x_2 = \omega_0^2\mu x_2$
(b) $\ddot{x}_1 + \omega_0^2 x_1 = \omega_0^2\mu x_2, \ddot{x}_2 + \omega_0^2 x_2 = \omega_0^2\mu x_1$
(c) $\ddot{x}_1 + \omega_0^2 x_1 = \omega_0^2\mu x_1, \ddot{x}_2 + \omega_0^2 x_2 = \omega_0^2\mu x_2$
(d) $\ddot{x}_1 + \omega_0^2 x_1 = \omega_0^2\mu x_1, \ddot{x}_2 + \omega_0^2 x_2 = \omega_0^2\mu x_1$

6. The normal modes of the system are:
(a) $\omega_0\sqrt{\mu^2 - 1}, \omega_0\sqrt{\mu^2 + 1}$
(b) $\omega_0\sqrt{1 - \mu^2}, \omega_0\sqrt{1 + \mu^2}$
(c) $\omega_0\sqrt{\mu - 1}, \omega_0\sqrt{\mu + 1}$
(d) $\omega_0\sqrt{1 - \mu}, \omega_0\sqrt{1 + \mu}$

7. A particle of mass m is moving in a potential of the form $V(x, y, z) = (1/2)\, m\omega^2 (3x^2 + 3y^2 + 2z^2 + 2xy)$. The oscillation frequencies of the three normal modes of the particle are given by:
(a) $\omega, \sqrt{3}\omega$ and $\sqrt{3}\omega$
(b) $\sqrt{2}\omega, \sqrt{3}\omega$ and $\sqrt{3}\omega$
(c) $\sqrt{2}\omega, \sqrt{2}\omega$ and 2ω
(d) $\sqrt{2}\omega, 2\omega$ and 2ω

8. The Lagrangian for a three particle system is given by $L = \frac{1}{2}\left(\dot{\eta}_1^2 + \dot{\eta}_2^2 + \dot{\eta}_3^2\right) - a^2\left(\eta_1^2 + \eta_2^2 + \eta_3^2 - \eta_1\eta_3\right)$, where a is real. Then one of the normal cooridnates has a frequency ω given by:

(a) $\omega^2 = a^2$ (b) $\omega^2 = \frac{a^2}{2}$ (c) $\omega^2 = 2a^2$ (d) $\omega^2 = \sqrt{2}a^2$

9 Two object each of mass m are connected by a spring of spring constant k and are capable of moving in a line. The normal modes have the frequencies.

(a) $\sqrt{\frac{2k}{m}}$ and 0 (b) $\sqrt{\frac{k}{m}}$ and 0 (c) $\sqrt{\frac{2k}{m}}$ and $\sqrt{\frac{k}{m}}$ (d) $\sqrt{\frac{k}{2m}}$ and 0

10. A particle of unit mass moves along the x-axis under the influence of a potential, $V(x) = x(x-2)^2$. The particle is found to be in stable equilibrium at the point $x = 2$. The time period of oscillation of the particle is

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) 2π **[GATE 2012]**

11. Consider the motion of a classical particle in a one dimensional double-well potential $V(x) = \frac{1}{4}\left(x^2 - 2\right)^2$. If the particle is displaced infinitesimally from the minimum the positive x-axis (and friction is neglected), then **[NET June 2012]**

(a) the particle will execute simple harmonic motion in the right well with an angular frequency $\omega = \sqrt{2}$
(b) the particle will execute simple harmonic motion in the right well with an angular frequency $\omega = 2$
(c) the particle will switch between the right and left wells
(d) the particle will approach the bottom of the right well and settle there

12. A particle of mass m is moving in the potential $V(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4$ where a, b are positive constants. The frequency of small oscillations about a point of stable equilibrium is **[NET Dec. 2014]**

(a) $\sqrt{a/m}$ (b) $\sqrt{2a/m}$ (c) $\sqrt{3a/m}$ (d) $\sqrt{6a/m}$

13. A particle of mass m moves in the one-dimensional potential $V(x) = \frac{\alpha}{3}x^3 + \frac{\beta}{4}x^4$ where $\alpha, \beta > 0$. One of the equilibrium points is $x = 0$. The angular frequency of small oscillations about the other equilibrium point is **[NET June 2015]**

(a) $\frac{2\alpha}{\sqrt{3m\beta}}$ (b) $\frac{\alpha}{\sqrt{m\beta}}$ (c) $\frac{\alpha}{\sqrt{12m\beta}}$ (d) $\frac{\alpha}{\sqrt{24m\beta}}$

14. A particle of mass 'm' is in a potential given by $V(r) = -\frac{a}{r} + \frac{ar_0^2}{3r^3}$ where 'a' and 'r_0' are positive constants. When distrubed slightly from its stable equilibrium position, it undergoes a simple harmonic oscillation. The time period of oscillation is **[GATE 2014]**

(a) $2\pi\sqrt{\frac{mr_0^3}{2a}}$ (b) $2\pi\sqrt{\frac{mr_0^3}{a}}$ (c) $2\pi\sqrt{\frac{2mr_0^3}{a}}$ (d) $4\pi\sqrt{\frac{mr_0^3}{a}}$

15. A particle of mass m moves in one dimension under the influence of a potential energy **[TIFR 2013]**

$$V(x) = -a\left(\frac{x}{\ell}\right)^2 + b\left(\frac{x}{\ell}\right)^4$$

where a and b are positive constants and ℓ is a characteristic length. The frequency of small oscillations about a point of stable equilibrium is

(a) $\frac{1}{2\pi\ell}\sqrt{\frac{b}{m}}$ (b) $\frac{1}{\pi\ell}\sqrt{\frac{a}{m}}$ (c) $\frac{1}{\pi\ell}\sqrt{\frac{a^2}{mb}}$ (d) $\frac{2b}{\pi\ell}\sqrt{\frac{1}{ma}}$

16. A particle of unit mass moves in a potential $V(x) = ax^2 + \frac{b}{x^2}$, where a and b are positive constants. The angular frequency of small oscillations about the minimum of the potential is: **[NET June 2011]**

(a) $\sqrt{8b}$ (b) $\sqrt{8a}$ (c) $\sqrt{8a/b}$ (d) $\sqrt{8b/a}$

17. The Lagrangian of a diatomic molecule is given by $L = \frac{m}{2}\left(\dot{x}_1^2 + \dot{x}_2^2\right) - \frac{k}{2}x_1x_2$, where m is the mass of each of the atoms and x_1 and x_2 are the displacements of atoms measured from the equilibrium position and $k > 0$. The norrmal frequencies are **[GATE 2009]**

(a) $\left(\frac{k}{m}\right)^{1/2}$ (b) $\left(\frac{k}{m}\right)^{1/4}$ (c) $\left(\frac{k}{2m}\right)^{1/4}$ (d) $\left(\frac{k}{2m}\right)^{1/2}$

18. A particle is placed in a region with the potential $V(x) = \frac{1}{2}kx^2 - \frac{\lambda}{3}x^3$, where $k, \lambda > 0$ then, **[GATE 2010]**

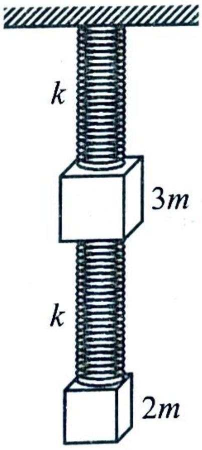
(a) $x = 0$ and $x = \frac{k}{\lambda}$ are points of stable equilibrium

(b) $x = 0$ is a point of stable equilbrium and $x = \frac{k}{\lambda}$ is a point of unstable equilibrium

(c) $x = 0$ and $x = \frac{k}{\lambda}$ are points of unstable equilibrium

(d) there are no points of stable or unstable equilibrium

19. Two masses $3m$ and $2m$ are suspended vertically by identical massless springs, each of stiffness constant k. The mass $2m$ is suspended from the mass $3m$ and the mass $3m$ is suspeded from a rigid support, as shown in the figure. If only vertical motion is permitted, the frequencies of small oscillations of this system are **[TIFR 2017]**

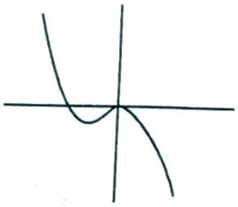
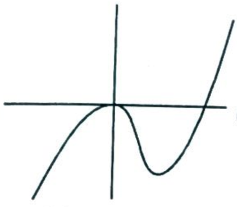
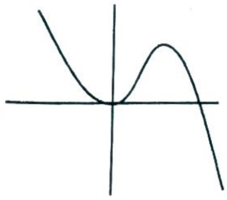
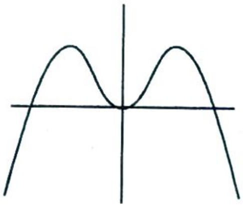


(a) $\sqrt{\frac{k}{m}}, \sqrt{\frac{k}{6m}}$ (b) $\sqrt{\frac{k}{2m}}, \sqrt{\frac{k}{3m}}$ (c) $\sqrt{\frac{k}{m}}, \sqrt{\frac{3k}{2m}}$ (d) $\sqrt{\frac{2k}{3m}}, \sqrt{\frac{3k}{2m}}$

20. A particle of mass 'm' is moving in a potential, $V(x)=\frac{1}{2}m\omega_0^2x^2+\frac{a}{2mx^2}$. What is minimum potential energy of the particle.

(a) $\omega_0\sqrt{a}$ (b) $2\omega_0\sqrt{a}$ (c) $\frac{\omega_0\sqrt{a}}{2}$ (d) $2\omega_0 a$

21. A particle of mass $\frac{2}{3}kg$ is subjected to a potential $V(x)=3x^2-2x^3$. Which of the following $V(x)$ versus x plot is correct?

(a)

(b)

(c)

(d)


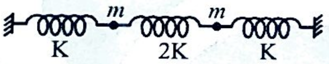
22. In the previous question if particle is released at $x=\frac{4}{3}$, what is its velocity when it reaches point $x=\frac{3}{2}$

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{1}{3}$ (d) $\frac{3}{2}$

23. In **Q.22**, what is maximum value of potential energy.

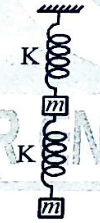
(a) 0.2 (b) 1 (c) 2 (d) 0

24. Two small particles are connected to each other and to the walls as shown in the figure frequencies of normal modes are



(a) $\sqrt{\frac{k}{m}},\sqrt{\frac{3k}{m}}$ (b) $\sqrt{\frac{3k}{m}},\sqrt{\frac{5k}{m}}$ (c) $\sqrt{\frac{k}{m}},\sqrt{\frac{k}{m}}$ (d) $\sqrt{\frac{k}{m}},\sqrt{\frac{2k}{m}}$

25. Frequency of normal mode for the system shown in the figure is



(a) $\sqrt{\left(3\pm\sqrt{2}\right)\frac{k}{m}}$ (b) $\sqrt{\left(3\pm\sqrt{5}\right)\frac{k}{m}}$ (c) $\sqrt{\left(5\pm\sqrt{3}\right)\frac{k}{m}}$ (d) $\sqrt{\left(3\pm\sqrt{3}\right)\frac{k}{m}}$

Answer Key

1. (c) 2. (b) 3. (c) 4. (a) 5. (b) 6. (d) 7. (c)
8. (d) 9. (a) 10. (b) 11. (b) 12. (b) 13. (b) 14. (a)
15. (b) 16. (b) 17. (d) 18. (b) 19. (a) 20. (a) 21. (c)
22. (b) 23. (b) 24. (c) 25. (b)

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Assignment-06: Classical Mechanics (CT, BP & GF)

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1. Which one of the following options is correct

(a) $\{q_i, q_j\} = 0, \{p_i, p_j\} = \delta_{ij}, \{q_i, p_j\} = \delta_{ij}$

(b) $\{q_i, q_j\} = 0, \{p_i, p_j\} = 0, \{q_i, p_j\} = \delta_{ij}$

(c) $\{q_i, q_j\} = \delta_{ij}, \{p_i, p_j\} = 0, \{q_i, p_j\} = 0$

(d) $\{q_i, q_j\} = \delta_{ij}, \{p_i, p_j\} = \delta_{ij}, \{q_i, p_j\} = 0$

2. $[p_x, e^{\alpha x^2}]$ is equal to

(a) $e^{\alpha x}$ (b) $2\alpha x e^{\alpha x^2}$ (c) $2\alpha x p_x e^{\alpha x^2}$ (d) $-2\alpha x e^{\alpha x^2}$

3. Poisson bracket $\{e^{\alpha q_i}, e^{\beta p_j}\}$ is equal to

(a) $e^{\alpha q_i} e^{\beta q_j}$ (b) $\alpha\beta e^{\alpha q_i} e^{\beta q_j}$ (c) $\alpha\beta\, e^{\alpha q_i} e^{\beta p_j} \delta_{ij}$ (d) zero

4. For a simple harmonic oscillator the Lagrangian is given by $L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}q^2$. If $A(p,q) = \frac{p+iq}{\sqrt{2}}$ and $H(p,q)$ is the Hamiltonian of the system, the Poisson bracket $\{A(p,q), H(p,q)\}$ is given by **[GATE 2008]**

(a) $iA(p,q)$ (b) $A^*(p,q)$ (c) $-iA^*(p,q)$ (d) $-iA(p,q)$

5. Lagrangian of a system is $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{qB}{2}(x\dot{y} - y\dot{x})$ for this system value of Poisson bracket $[\dot{x}, \dot{y}]$ is equal to

(a) $\frac{qB}{2m^2}$ (b) $\frac{qB}{m^2}$ (c) $\frac{2qB}{m^2}$ (d) zero

6. What is the Poisson bracket between the position x and x-component of the angular momentum L_x

(a) x (b) y (c) z (d) 0

7. If P_x and L_z are constants of motion, then their Poisson bracket $[P_x, L_z]$ is :

(a) Also a constant of motion (b) Not a constant of motion.

(c) Zero (d) Undefined.

8. $PB\{(\vec{r}\cdot\vec{p}), |\vec{r}|\}$ is equal to

(a) $-|\vec{p}|$ (b) $-|\vec{r}|$ (c) $|\vec{r}|$ (d) $|\vec{p}|$

9. $PB\{r^2, p^2\}$ is equal to

(a) $2rp$ (b) $2\vec{r}\cdot\vec{p}$ (c) $4\vec{r}\cdot\vec{p}$ (d) $4rp$

10. $PB\{\vec{a}\cdot\vec{r}, \vec{b}\cdot\vec{p}\}$ is equal to

(a) ab (b) $\vec{a}(\vec{r}\cdot\vec{b})$ (c) $\vec{b}(\vec{r}\cdot\vec{a})$ (d) $\vec{a}\cdot\vec{b}$

11. $PB\{\vec{r},\{\vec{a}\cdot\vec{p},\ \vec{b}\cdot\vec{L}\}\}$ is equal to

(a) $\vec{a}\times\vec{b}$ (b) $\vec{b}\times\vec{a}$ (c) $\vec{a}\cdot\vec{b}$ (d) $(\vec{a}\times\vec{b})\cdot\vec{L}$

12. $PB\{\vec{p},\{\vec{a}\cdot\vec{r},\ \vec{b}\cdot\vec{p}\}\}$ is equal to

(a) $\vec{a}\times\vec{b}$ (b) 0 (c) $\vec{a}\cdot\vec{b}$ (d) $(\vec{a}\times\vec{b})\cdot\vec{p}$

13. $PB[\vec{r}\cdot\vec{p},\vec{L}]$ is equal to

(a) 1 (b) 0 (c) $\vec{r}$ (d) $\vec{p}$

14. $PB[\vec{r}\cdot\vec{L},\ \vec{p}]$ is equal to

(a) $2\vec{L}$ (b) $2\vec{p}$ (c) $2\vec{r}$ (d) zero

15. $PB[\vec{p}\cdot\vec{L},\ \vec{r}]$ is equal to

(a) $-2\vec{L}$ (b) $-2\vec{p}$ (c) $-2\vec{r}$ (d) zero

16. $PB[L^2,\ \vec{r}\cdot\vec{p}]$ is equal to

(a) $4L^2$ (b) $-4L^2$ (c) 4 (d) zero

17. Value of poisson bracket $[\vec{a}.\vec{L},\vec{b}.\vec{r}]$ is

(a) $(\vec{a}\times\vec{b}).\vec{r}$ (b) $(\vec{b}\times\vec{a}).\vec{r}$ (c) $(\vec{a}\times\vec{b}).\vec{p}$ (d) $(\vec{b}\times\vec{a}).\vec{p}$

18. Value of Poisson bracket is $\{r^2,\ p^2\}$ is

(a) $2rp$ (b) $4\vec{r}\cdot\vec{p}$ (c) 4 (d) $4rp$

19. If L_z be z component of angular momentum and p_y be y component of linear momentum than $[L_z, xp_y]$ is equal to

(a) $yp_y + xp_x$ (b) $yp_y - xp_x$ (c) $xp_x - yp_y$ (d) zero

20. Poisson bracket $[L_x, yL_z]$ is equal to

(a) zL_z (b) $p_z L_z$ (c) $zL_z - yL_y$ (d) $zL_z + yL_y$

21. Two sets of transformation of conjugate position and momentum are given as (i) Q = p, P = – q and (ii) Q = p + iq, P = p–iq. Which of the following statements is correct?

(a) Both (i) and (ii) represent canonical transformations.
(b) (i) represent a canonical transformation but (ii) does not represents a canonical transformation.
(c) (i) does not represent a canonical transformation but (ii) does not representsa canonical transformation.
(d) Neither (i) nor (ii) represents any canonical transformation.

22. If the transformation $Q = aq + bp,\ P = cq + dp$ is canonical then

(a) ab + cd = 1 (b) ac + bd = 1 (c) ad – bc = 1 (d) ab – cd = 1

23. If the following transformation is canonical

$$Q = q\tan\alpha p,\quad P = 2\ln\sin\beta p$$

values of α and β are respectively.

(a) (1, 1) (b) (1, ½) (c) (½, 1) (d) ½, ½

24. A linear transformation of a generalized coordinate q and the corresponding momentum p to Q and P given by

$$Q = q + p;\ \ P = q + \alpha p$$

is canonical if the value of the constant α is:

(a) -1 (b) 0 (c) $+1$ (d) $+2$

25. Let (p, q) and (P, Q) be two pairs of canonical variables. The transformation **[GATE 2011]**

$$Q = q^{\alpha}\cos(\beta p);\ P = q^{\alpha}\sin(\beta p)$$

is canonical for

(a) $\alpha = 2, \beta = 1/2$ (b) $\alpha = 2, \beta = 2$ (c) $\alpha = 1, \beta = 1$ (d) $\alpha = 1/2, \beta = 2$

26. Canonical transformation defined by the generating function, $F(Q, p) = -\left(e^{Q} - 1\right)^{2}\tan p$ is

(a) $Q = \ln\left[\sqrt{q}\cos p + 1\right],\ P = 2\sqrt{q}\left[\sqrt{q}\cos p + 1\right]\sin p$

(b) $Q = 2\sqrt{q}\left[\sqrt{q}\cos p + 1\right]\sin p,\ P = \sqrt{q}\left[\sqrt{q}\cos p + 1\right]$

(c) $Q = \ln\left[\sqrt{q}\sin p + 1\right],\ P = \sqrt{q}\left[\sqrt{q}\sin p + 1\right]\cos p$

(d) $Q = \ln\left[\sqrt{q}\sin p + 1\right],\ P = 2\sqrt{q}\left[\sqrt{q}\sin p + 1\right]\cos p$

27. The generating function $F(P, q)$ for the transformation $p = \frac{1}{Q}, q = PQ^{2}$ is

(a) $\sqrt{Pq}$ (b) $-\sqrt{Pq}$ (c) $2\sqrt{Pq}$ (d) $-2\sqrt{Pq}$

28. Consider a canonical transformation $(q,\ p) \to (Q,\ P)$ such that $Q = \log(1 + q^{1/2}\cos p)$

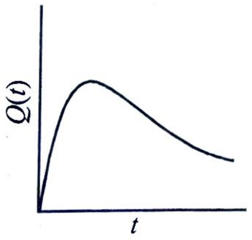
$$P = 2(1 + q^{1/2}\cos p)q^{1/2}\sin p$$

generating function is

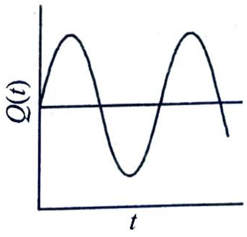
(a) $(e^{Q} - 1)^{2}\tan p$ (b) $-(e^{Q} - 1)^{2}\tan p$ (c) $(e^{Q} - 1)\tan p$ (d) $-(e^{Q} - 1)\tan p$

29. A canonical transformation $(p, q) \to (P, Q)$ is performed on the Hamiltonian $H = \frac{1}{2m}p^{2} + \frac{1}{2}m\omega^{2}q^{2}$ via the generating function $F = \frac{1}{2}m\omega q^{2}\cot Q$. If $Q(0) = 0$, which of the following graphs shows schematically the dependence of $Q(t)$ on t? **[NET Dec. 2015]**

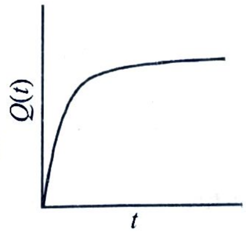
(a)



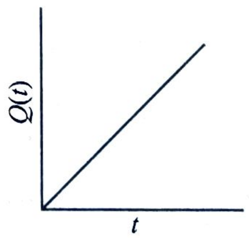
(b)



(c)



(d)



30. The Hamilton's canonical equations of motion in terms of Poisson Brackets are **[GATE 2014]**

(a) $\dot{q}=\{q,H\};\ \dot{p}=\{p,H\}$ (b) $\dot{q}=\{H,q\};\ \dot{p}=\{H,q\}$

(c) $\dot{q}=\{H,p\};\ \dot{p}=\{H,q\}$ (d) $\dot{q}=\{p,H\};\ \dot{p}=\{q,H\}$

31. A mechanical system is described by the Hamiltonian $H(q,p)=\frac{p^2}{2m}+\frac{1}{2}m\omega^2q^2$. As a result of the canonical transformation generated by $F(q,Q)=-\frac{Q}{q}$, the Hamiltonian in the new coordinate Q and momentum P becomes **[NET Dec. 2014]**

(a) $\frac{1}{2m}Q^2P^2+\frac{m\omega^2}{2}Q^2$ (b) $\frac{1}{2m}Q^2P^2+\frac{m\omega^2}{2}P^2$

(c) $\frac{1}{2m}P^2+\frac{m\omega^2}{2}Q^2$ (d) $\frac{1}{2m}Q^2P^4+\frac{m\omega^2}{2}P^{-2}$

32. A canonical transformation $(q,\,p)\to(Q,\,P)$ is made through the generating function $F(q,\,P)=q^2P$ on the Hamiltonian $H(q,\,p)=\frac{p^2}{2\alpha q^2}+\frac{\beta}{4}q^4$, where α and β are constants. The equations of motion for $(Q,\,P)$ are **[NET June 2016]**

(a) $\dot{Q}=\frac{P}{\alpha}$ and $\dot{P}=-\beta Q$ (b) $\dot{Q}=\frac{4P}{\alpha}$ and $\dot{P}=\frac{-\beta Q}{2}$

(c) $\dot{Q}=\frac{P}{\alpha}$ and $\dot{P}=-\frac{2P^2}{Q}-\beta Q$ (d) $\dot{Q}=\frac{2P}{\alpha}$ and $\dot{P}=-\beta Q$

33. If p and q are the position and momentum variables, which one of the following is NOT a canonical transformation? **[GATE 2009]**

(a) $Q=\alpha q$ and $P=\frac{1}{\alpha}p$, for $\alpha\neq 0$

(b) $Q=\alpha q+\beta p$ and $P=\beta q+\alpha p$ for α,β real and $\alpha^2-\beta^2=1$

(c) $Q=p$ and $P=q$

(d) $Q=p$ and $P=-q$

34. The Poisson bracket $\left[x,\,xp_y+yp_x\right]$ is equal to **[GATE 2017]**

(a) $-x$ (b) y (c) $2p_x$ (d) p_y

35. If $A=\frac{p^2}{2}-\frac{1}{2q^2}$, $B=\frac{pq}{2}$, Poisson bracket $\{B,A\}$ is equal to

(a) B (b) zero (c) A (d) –A

36. Value of Poisson bracket $\{\{L_i, x_j\}, \{p_i, L_j\}\}$ is (All the symbols have usual meaning)

(a) 0 (b) 6 (c) −6 (d) 2

Answer Key

1. (b) **2. (d)** **3. (c)** **4. (a)** **5. (b)** **6. (d)** **7. (a)**
8. (b) **9. (c)** **10. (d)** **11. (a)** **12. (b)** **13. (c)** **14. (d)**
15. (d) **16. (d)** **17. (a)** **18. (b)** **19. (b)** **20. (c)** **21. (b)**
22. (c) **23. (d)** **24. (d)** **25. ()** **26. (a)** **27. (c)** **28. (b)**
29. (d) **30. (a)** **31. (d)** **32. (b)** **33. (c)** **34. (b)** **35. (c)**
36. (b)

CSIR-UGC-NET/JRF | GATE PHYSICS

Assignment-07: Classical Mechanics (Special Theory of Relativity)

1. Let (x, t) and (x', t') be the coordinate systems used by the observers O and O', respectively. Observer O' moves with a velocity $v = \beta c$ along their common positive x-axis. If $x_+ = x + ct$ and $x_- = x - ct$ are the linear combinations of the coordinates, the Lorentz transformation relating O and O' takes the form

(a) $x'_+ = \dfrac{x_- - \beta x_+}{\sqrt{1-\beta^2}}$ and $x'_- = \dfrac{x_+ - \beta x_-}{\sqrt{1-\beta^2}}$, (b) $x'_+ = \sqrt{\dfrac{1+\beta}{1-\beta}}\, x_+$ and $x'_- = \sqrt{\dfrac{1-\beta}{1+\beta}}\, x_-$ **[NET June 2016]**

(c) $x'_+ = \dfrac{x_+ - \beta x_-}{\sqrt{1-\beta^2}}$ and $x'_- = \dfrac{x_- - \beta x_+}{\sqrt{1-\beta^2}}$, (d) $x'_+ = \sqrt{\dfrac{1-\beta}{1+\beta}}\, x_+$ and $x'_- = \sqrt{\dfrac{1+\beta}{1-\beta}}\, x_-$

2. In an inertial farme S, two events A and B take place at $(ct_A = 0, \vec{r}_A = 0)$ and $(ct_B = 0, \vec{r}_B = 2\hat{y})$, respectively. The times at which these events take place in a frame $S¢$ moving with a velocity $0.6\,c\,\hat{y}$ with respect to S are given by **[GATE 2015]**

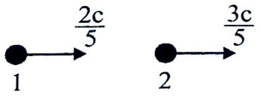
(a) $ct'_A = 0;\ ct'_B = -3/2$ (b) $ct'_A = 0;\ ct'_B = 0$

(c) $ct'_A = 0;\ ct'_B = 3/2$ (d) $ct'_A = 0;\ ct'_B = 1/2$

3. A light beam is propagating through a medium with index of refraction 1.5. If the medium is moving at constant velocity $0.7c$ in the same direction as the beam, what is the velocity of light in the medium as measured by an observer in the laboratory ? (c = velocity of light in vacuum)

[✓ (a) $0.93c$] (b) $0.98c$ (c) $0.96c$ (d) $0.9c$

4. Two electrons are moving with speeds $\dfrac{2c}{5}$ and $\dfrac{3c}{5}$ as shown in the figure. Velocity of 2 with respect to 1 is

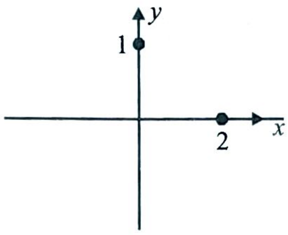


[✓ (a) $\dfrac{5c}{19}$] (b) $\dfrac{-5c}{19}$ (c) $\dfrac{c}{5}$ (d) $-\dfrac{c}{5}$

5. In the previous question what is velocity of ~~approach~~ Separation as seen by an observer on ground.

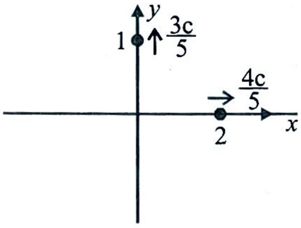
(a) c (b) $\dfrac{c}{5}$ (c) $\dfrac{5c}{19}$ (d) $\dfrac{2c}{3}$

6. Two photons are moving as shown in the figure. Velocity of 1 with respect to 2 is



(a) $c(\hat{i}+\hat{j})$ (b) $c(-\hat{i}+\hat{j})$ (c) $-c\hat{i}$ (d) $c\hat{j}$

7. Two electrons are moving as shown in the figure. Velocity of 1 with respect to 2 is



(a) $\frac{12c}{13}$ (b) $\frac{20c}{25}$ (c) $\frac{\sqrt{481}}{25}c$ (d) c

8. Two particles are moving with constant speeds equal to $\frac{C}{2}$ along +X and +Y direction respectively. Magnitude of their relative velocity is

(a) $\frac{C}{2}$ (b) $\frac{\sqrt{7}}{5}C$ (c) $\frac{\sqrt{7}}{4}C$ (d) $\frac{\sqrt{7}}{8}C$

9. A beam of light propagates in a slab of glass of refractive index n in the positive x-direction. The slab itself is also moving in the positive direction with a speed v in laboratory frame. What is the speed of the beam of light as measured in the laboratory frame ?

(a) c (b) $(c^2n+cv)/(c+nv)$

(c) $c\left(1-\frac{1}{n}\right)$ (d) $(c^2+vcn)/(cn+v)$

10. A circle of radius 5m lies at rest in x – y plane in the laboratory. For an observer moving with a uniform velocity υ along the y direciton, the circle appears to be an ellipse with an equation $\frac{x^2}{25}+\frac{y^2}{9}=1$.

The speed of the observer in terms of the velocity of light c is:

(a) 9c/25 (b) 3c/5 (c) 4c/5 (d) 16c/25

11. A square loop of area A_0(when at rest) is moving with speed $\frac{C}{\sqrt{5}}$ in a direction parallel to one of its diagonal area of the moving loop is

(a) A_0 (b) $\sqrt{\frac{3}{5}}A_0$ (c) $\sqrt{\frac{4}{5}}A_0$ (d) $\sqrt{\frac{9}{10}}A_0$

12. Apparent length of a rod is half of its proper length. If direction of velocity of the rod makes 30° with the length of the rod then velocity of rod is equal to

(a) C (b) C/2 (c) $\frac{\sqrt{3}}{2}C$ (d) $\frac{2C}{3}$

13. A cube of side 'a' is moved with speed $\frac{4c}{5}$ along it body diagonal apparent volume of the cube is

(a) $\frac{3a^3}{10}$ (b) $\frac{3a^3}{5}$ (c) $\sqrt{\frac{33}{75}}a^3$ (d) $\left(\frac{59}{75}\right)^{3/2}a^3$

14. A rod of proper length l_0 oriented parallel to the x-axis moves with speed u along the x-axis is S. What is length measured by an observer in S' which is moving along $+x$ direction w.r.t. S with speed v.

(a) $l_0\sqrt{1-\frac{v^2}{c^2}}\sqrt{1+\frac{u^2}{c^2}}$ (b) $\frac{l_0\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{u^2}{c^2}}}{1-\frac{uv}{c^2}}$ (c) $\frac{l_0\sqrt{1+\frac{v^2}{c^2}}\sqrt{1+\frac{u^2}{c^2}}}{1+\frac{uv}{c^2}}$ (d) $l_0\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{u^2}{c^2}}$

15. In an inertial frame S, a stationary rod makes an angle θ with the x-axis. Another inertial frame S' moves with a velocity 'v' with respect ot S along the common $x-x'$ axis. As observed from S' the angle made by the rod with the x'– axis is θ'. Which of the following statements is correct?

(a) $\theta' < \theta$

(b) $\theta' > \theta$

(c) $\theta' < \theta$ if v is negative and $\theta' > \theta$ if v is positive

(c) $\theta' > \theta$ if v is negative and $\theta' < \theta$ if v is positive.

16. If the half-life of an elementary particle moving with speed 0.9c in the laboratory frame is 5×10^{-8}s, then the proper half-life is ______ $\times10^{-8}$ s. ($c = 3\times10^{8}$ m/s).

17. The distance of a star from the Earth is 4.25 light years, as measured from the Earth. A space ship travels from Earth to the star at a constant velocity in 4.25 years, according to the clock on the ship. The speed of the space ship in units of the speed of light is, **[JEST 2015]**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{2}{3}$ (d) $\frac{1}{\sqrt{3}}$

18. A relativistic particle moves with a constant velocity υ with respect to the laboratory frame. In time τ, measured in the rest frame of the particle, the distance that it travels in the laboratory frame is

(a) $\upsilon\tau$ (b) $\frac{c\tau}{\sqrt{1-\frac{\upsilon^2}{c^2}}}$ (c) $\upsilon\tau\sqrt{1-\frac{\upsilon^2}{c^2}}$ (d) $\frac{\upsilon\tau}{\sqrt{1-\frac{\upsilon^2}{c^2}}}$ **[NET Dec. 2016]**

19. Life time of a muon in laboratory frame is 2.5×10^{-6} sec. Muon flies with speed $\frac{3C}{5}$. What is life time of muon in its rest frame.

(a) 2×10^{-6} sec (b) 1.5×10^{-6} sec (c) 1×10^{-6} sec (d) 2.5×10^{-6} sec

20. Proper life time of a muon is 2×10^{-6} sec. It is travelling with speed $\frac{4c}{5}$. What is the distance moved past the muon as seen by the muon.

(a) 480 meter (b) 288 meter (c) 384 meter (d) 256 meter

21. A Radio active particle has proper mean life of 1μsec moves through the laboratory at a speed 2.7×10^{8} meter/sec. What will be its lifetime observed by an observer in laboratory.

(a) 10 μs (b) 0.1μs (c) 2.30μs (d) 0.43μs

22. A relativistic particle of rest mass m_0 is moving with a speed v. The value of v at which its kinetic energy is equal to its rest energy is:

(a) $v=\frac{c}{2}$ (b) $v=\frac{c}{4}$ (c) $v=\frac{\sqrt{3}}{2}c$ (d) $v=2\left(\sqrt{2}-1\right)c$

23. A meter rod is moving along the positive x axis with velocity 0.6C. Its length as measured by a stationary observer in meters, is:

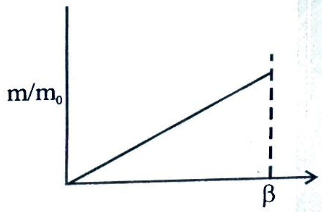
(a) 0.6 (b) 0.8 (c) 1.0 (d) 1.2

24. A particle moves with a relativistic speed such that its energy is twice the rest mass energy. The speed of the particle is:

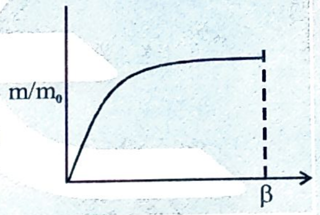
(a) $v=\frac{c}{2}$ (b) $v=\frac{3}{4}c$ (c) $v=\frac{\sqrt{3}}{2}c$ (d) $v=c$

25. For a particle moving with relativistic speed, the ratio of its relativistic mass to its rest mass (m/m_0) varies with the parameter $\beta=v/c$ as

(a)



(b)



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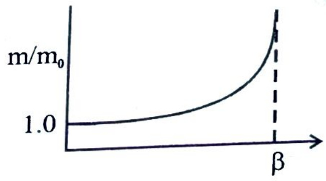

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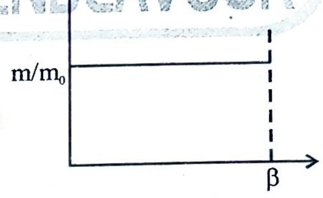


26. A particle of mass M at rest decays into two identical particles of mass m each such that 2m is less than M. The speed of each particle after decay

(a) $v=0$ (b) $v=c\sqrt{1+(2m/M)^2}$ (c) $v=c\sqrt{M/m}$ (d) $v=c\sqrt{1-(2m/M)^2}$

27. The mass m of a moving particle is $\frac{2m_0}{\sqrt{3}}$, where m_0 is its rest mass. The linear momentum of the particle is:

(a) $2m_0c$ (b) $\frac{2m_0c}{\sqrt{3}}$ (c) $\frac{m_0c}{\sqrt{3}}$ (d) m_0c

28. An electron gains energy so that it mass becomes $2m_0$. Its speed is:

(a) $\frac{\sqrt{3}}{2}c$ (b) $\frac{3}{4}c$ (c) $\frac{3}{2}c$ (d) $\sqrt{\frac{3}{2}}c$

29. The speed of a particle whose kinetic energy is equal to its rest mass energy is given by (c is the speed of light in vacuum)

(a) c/3 (b) $\sqrt{2}c/3$ (c) c/2 (d) $\frac{\sqrt{3}}{2}c$

30. The momentum of an electron (mass m) which has the same kinetic energy as its rest mass energy is (c is velocity of light)

(a) $\sqrt{3}$ mc (b) $\sqrt{2}$ mc (c) mc (d) $mc/\sqrt{2}$

31. Kinetic energy of a relativistic particle of rest mass m moving with speed v is:

(a) $\frac{1}{2}mv^2$ (b) $\frac{mc^2}{\sqrt{1-v^2/c^2}}$ (c) $\frac{mc^2}{\sqrt{1-v^2/c^2}}-mc^2$ (d) $\frac{1}{2}m\left(v^2-c^2\right)$

32. A particle has rest mass m_o, and momentum m_oc, where c is the velocity of light. The total energy and the velocity of the particle are respectively

(a) $\sqrt{2}m_oc^2$ and $c/2$ (b) $2m_oc^2$ and $c/\sqrt{2}$ (c) $\sqrt{2}m_oc^2$ and $c/\sqrt{2}$ (d) $2m_oc^2$ and $c/2$

H.W.

33. Let v, p and E denotes the speed, the magnitude of the momentum, and the energy of a free particle of rest mass 'm'. Then **[NET Dec. 2012]**

(a) $\frac{dE}{dp}=$ constant (b) $p=mv$ (c) $v=\frac{cp}{\sqrt{p^2+m^2c^2}}$ (d) $E=mc^2$

34. The velocity of a particle at which the kinetic energy is equal to its rest energy is (in terms of c, the speed of light in vacuum)

(a) $\sqrt{3}c/2$ (b) $3c/4$ (c) $\sqrt{3/5}c$ (d) $c/\sqrt{2}$

35. According to the special theory of relativity, the speed v of a free particle of mass m and total energy E is: **[NET Dec. 2014]**

(a) $v=c\sqrt{1-\frac{mc^2}{E}}$ (b) $v=\sqrt{\frac{2E}{m}\left(1+\frac{mc^2}{E}\right)}$ (c) $v=c\sqrt{1-\left(\frac{mc^2}{E}\right)^2}$ (d) $v=c\left(1+\frac{mc^2}{E}\right)$

36. A particle with rest mass M is at rest and decays into two particles of equal rest masses $\frac{3}{10}M$ which move along the z-axis. Their velocities are given by **[GATE 2015]**

(a) $\vec{v}_1=\vec{v}_2=(0.8c)\hat{z}$ (b) $\vec{v}_1=-\vec{v}_2=(0.8c)\hat{z}$

(c) $\vec{v}_1=-\vec{v}_2=(0.6c)\hat{z}$ (d) $\vec{v}_1=(0.6c)\hat{z};\ \vec{v}_2=(-0.8c)\hat{z}$

37. In an observer's rest frame, a particle is moving towards the observer with an energy E and momentum p. If c denotes the velocity of light in vacuum, the energy of the particle in another frame moving in the same direction as the particle with a constant velocity v is

(a) $\frac{(E+vp)}{\sqrt{1-(v/c)^2}}$ (b) $\frac{(E-vp)}{\sqrt{1-(v/c)^2}}$ (c) $\frac{(E+vp)}{[1-(v/c)^2]^2}$ (d) $\frac{(E-vp)}{[1-(v/c)^2]^2}$

38. The kinetic energy of a particle of rest mass m_0 is equal to its rest mass energy. Its momentum in units of m_0c, where c is the speed of light in vacuum, is _________. (Give your answer upto two decimal places)

[GATE 2016]

39. The kinetic energy of a free relativistic particle is defined as $E - m_0c^2$, where E and m_0 are respectively its total energy and rest mass. Let 'v' be the speed of the particle when its kinetic energy is half of its rest mass energy. Then the ratio v/c is

(a) $\frac{\sqrt{5}}{3}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\frac{\sqrt{3}}{2}$

40. Lagrangian of a relativistic particle is

(a) $m_0\sqrt{1-\frac{v^2}{c^2}}$ (b) $-m_0c^2\sqrt{1-\frac{v^2}{c^2}}$ (c) $\frac{m_0v^2}{2}$ (d) $\frac{m_0v^2}{2\sqrt{1-\frac{v^2}{c^2}}}$

41. An object of rest mass m_0 collides perfectly inelastically with another object kept at rest and having same rest mass. The rest mass of the combined system will be

(a) equal to $2m_0$ (b) greater than $2m_0$ (c) smaller than $2m_0$ (d) m_0

42. Length of a meter stick appears to be 50cm. How fast is it moving?

(a) $\frac{\sqrt{3}}{2}c$ (b) $\sqrt{3}\,c$ (c) $\frac{\sqrt{3}}{4}c$ (d) $\frac{c}{2}$

43. Rest mass of a relativistic particle can be expressed in terms of its momentum and K.E as

(a) $\frac{(pc)^2-T^2}{2Tc^2}$ (b) $\frac{(pc)^2+T^2}{2Tc^2}$ (c) $\frac{(pc)^2+T^2}{Tc^2}$ (d) $\frac{(pc)^2-T^2}{Tc^2}$

44. Energy equivalent of an electron is

(a) 5110 Kev (b) 511 Kev (c) 51100 Kev (d) 511000 Kev

45. Planck's constant (h), the speed of light in vacuum (c) and Newton's gravitational constant (G) are three fundamental constants of nature. Which of the following combinations has the dimensions of length?

(a) $\sqrt{hG}\,/\,c^{3/2}$ (b) $\sqrt{hG}\,/\,c^{5/2}$ (c) $\sqrt{hc/G}$ (d) $\sqrt{Gc}\,/\,h^{3/2}$

46. Dynamic mass of a particle is measured to be $\sqrt{3}$ times its rest mass. Ratio of kinetic energy and total energy is

(a) $\frac{\sqrt{5}-1}{\sqrt{5}}$ (b) $\frac{\sqrt{3}-1}{\sqrt{3}}$ (c $\frac{\sqrt{2}-1}{\sqrt{2}}$ (d) $\frac{\sqrt{7}-1}{\sqrt{7}}$

47. A source of light is moving towards an observer with speed $v \ll c$. Value of $\frac{\Delta\lambda}{\lambda_0}$ is

(a) $\frac{2v}{c}$ (b) $\frac{v}{c}$ (c) $\frac{v}{2c}$ (d) $\frac{v^2}{c^2}$

48. A pion moving with speed 3c/5 emits a muon. As seen in the lab frame muon moves perpendicular to pion with speed 4c/5. With what speed relative to pion the muon is emitted.

(a) $\frac{\sqrt{481}}{13}c$ (b) $\frac{\sqrt{481}}{25}c$ (c) $\frac{10c}{17}$ (d) $\frac{2c}{5}$

49. Two photons approaches each other. The relative velocity of them will be

(a) c/2 (b) c/3 (c) c/4 (d) c

50. A particle of mass 'm' which is at rest in the laboratory decays into two particles, one of which has mass 'm/2' while other is massless. The magnitude of the momentum of each of the particles in the laboratory frame will be

(a) 3mc/8 (b) mc/8 (c) 3mc/4 (d) mc/4

51. A constant force F is applied to a relativistics particle of rest mass m_0. If the particle starts from rest at $t=0$, its dynamic mass after a time 't' is

(a) $\frac{m_0}{\sqrt{1+\frac{F^2t^2}{m_0^2c^2}}}$ (b) $\frac{1}{c}\sqrt{F^2t^2+m_0^2c^2}$ (c) $\frac{m_0}{\sqrt{1-\frac{F^2t^2}{m_0^2c^2}}}$ (d) $m_0 \tan h\frac{Ft}{m_0c}$

52. Which one of the following quantities is invariant under Lorentz transformation? **[GATE 2014]**

(a) Charge density ~~(b)~~ Charge (c) Current (d) Electric field

53. Which of the following is invariant under Lorentz transformation

(a) $\frac{p}{E}$ (b) $\frac{p^2}{E}$ (c) $\frac{d^3\vec{p}}{E}$ (d) $Ed^3\vec{p}$

54. A person is sitting at the centre of a circle. And a light source emitting a light of wavelength 589 nm is moving along the circle with speed 3c/5. What is wavelength of the light observed by the person

(a) 736 nm (b) 393 nm (c) 1178 nm (d) 295 nm

55. A galaxy is moving away from the earth at such a speed that blue light $(\lambda = 400nm)$ appears to be $\lambda = 600\ nm$. The speed of galaxy is

(a) $\frac{3c}{5}$ (b) $\frac{5c}{13}$ (c) $\frac{5c}{9}$ (d) $\frac{4c}{9}$

56. In lab frame two events occur simultaneously. An observer moving along the line joining the two events with velocity v_1 with respect to lab finds them to occur at a time difference Δt_1 while another observer moving in same direction with velocity v_2 with respect to lab finds the time difference to be Δt_2. Value of $\frac{\Delta t_1}{\Delta t_2}$ is

(a) $\frac{v_1}{v_2}$ (b) $\frac{v_2}{v_1}$ (c) $\frac{v_1}{v_2}\sqrt{\frac{c^2-v_1^2}{c^2-v_2^2}}$ (d) $\frac{v_1}{v_2}\sqrt{\frac{c^2-v_2^2}{c^2-v_1^2}}$

57. Proper mean life time of an unstable radioactive particle moving lab frame is $4\mu s$ and it is found to travel a distance of 900 meter. Speed of the particle is

(a) 0.5c (b) 0.6c (c) 0.8c (d) 0.4c

58. Two events occur simultaneously in S-frame. In S'-frame which moves along the line joining the two events with velocity 0.6c with respect to S, the two events are separated by a distance 4×10^8 meter. Time interval between the two events in S' frame is

(a) 0.85 sec (b) 1.5 sec (c) 2 sec (d) 2.5 sec

59. A particle of rest mass m_0 is subjected to a constant force F due to which particle accelerates from rest after what time kinetic energy of the particle will become equal to its rest mass energy

(a) $\frac{m_0 c}{F}$ (b) $\frac{m_0 c}{2F}$ (c) $\frac{\sqrt{3}\, m_0 c}{2F}$ (d) $\frac{\sqrt{3}\, m_0 c}{F}$

Answer Key

1. (d)	2. (a)	3. (a)	4. (a)	5. (b)	6. (c)	7. (c)
8. (c)	9. (d)	10. (c)	11. (c)	12. (a)	13. (b)	14. (b)
15. (b)	16. (2.1–2.3)	17. (b)	18. (d)	19. (a)	20. (a)	21. (c)
22. (c)	23. (b)	24. (c)	25. (c)	26. (d)	27. (c)	28. (a)
29. (d)	30. (a)	31. (c)	32. (c)	33. (c)	34. (a)	35. (c)
36. (b)	37. (b)	38. (1.72 to 1.74)		39. (a)	40. (b)	41. (b)
42. (a)	43. (a)	44. (b)	45. (a)	46. (b)	47. (b)	48. (b)
49. (d)	50. (a)	51. (b)	52. (b)	53. (c)	54. (a)	55. (b)
56. (d)	57. (b)	58. (a)	59. (d)			

CSIR-UGC-NET/JRF | GATE PHYSICS

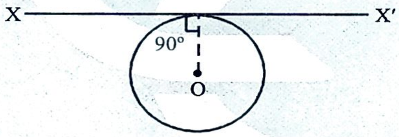
Assignment-08: Classical Mechanics (Rigid Body)

1. Particles of masses 1, 2, 3 kg respectively are arranged at the comes of an equilateral triangle of side 1 m. The M.I. of system about an axis passing through the centroid and perpendicular to the plane of triangle is
(a) 2 kg–m^2 (b) 3 kg–m^2 (c) 1 kg–m^2 (d) 6 kg–m^2.

2. The moment of inertia of a thin rectangular plate of length a, width b, mass m about an axis passing through the centre and perpendicular to the plate is
(a) $\frac{1}{12}m\left(a^2+b^2\right)$ (b) $\frac{1}{6}m\left(a^2+b^2\right)$ (c) $\frac{1}{12}mab$ (d) $\frac{1}{6}mab$

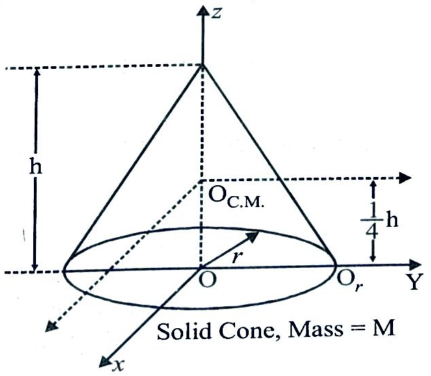
3. An annulus of mass M made of a material of uniform density has inner and outer radii 'a' and 'b' respectively. Its principal momnet of inertia along the axis of symmetry perpendicular to the plane of the annulus is :
(a) $\frac{1}{2}M\frac{\left(b^4+a^4\right)}{\left(b^2-a^2\right)}$ (b) $\frac{1}{2}M\pi\left(b^2-a^2\right)$ (c) $\frac{1}{2}M\left(b^2-a^2\right)$ (d) $\frac{1}{2}M\left(b^2+a^2\right)$

4. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is:



(a) $\frac{\rho L^3}{8\pi^2}$ (b) $\frac{\rho L^3}{16\pi^2}$ (c) $\frac{5\rho L^3}{16\pi^2}$ (d) $\frac{3\rho L^3}{8\pi^2}$

5. Consider the uniform solid right cone depicted in the figure on the right. This cone has mass M and a circular base of radius r. If the moment of inertia of the cone about an axis parallel to the X axis passing through the centre of mass $O_{C.M.}$ (see figure) is given by $\frac{3}{80}M(4r^2+h^2)$ then the moment of inertia about another axis parallel to the X axis, but passing through the point O_r (see figure), is **[TIFR 2013]**



(a) $\frac{3}{80}M(4r^2+h^2)$ (b) $\frac{3}{40}M(2r^2+h^2)$

(c) $\frac{1}{20}M(23r^2+2h^2)$ (d) $\frac{1}{30}M(15r^2+4h^2)$

6. Three identical rings of radius 'a' are placed on the xy-plane so that each ring touches the other two tangentially. Each ring, which may be considered to be infinitesimally thin, has a mass M. The centres of the three rings are equidistant from the z-axis— this common distance being $2a/\sqrt{3}$. What is the moment of inertia of this system about the z-axis?

(a) $3Ma^2/4$ (b) $3Ma^2/2$ (c) $6Ma^2$ (d) $7Ma^2$

7. Two circular discs A and B of equal mass and thickness but made of metals with densities d_A and d_B ($d_A > d_B$). If their moments of inertia about an axis passing through the centre and normal to the circular faces be I_A and I_B, then

(a) $I_A = I_B$ (b) $I_A > I_B$ (c) $I_A < I_B$ (d) $I_A \geq I_B$

8. The radius of gyration of a rod of mass 100 gm and length 100 cm about an axis passing through its centre of gravity and perpendicular to its length is given by

(a) $100/2\sqrt{3}$ (b) $50/3\sqrt{2}$ (c) $100/3\sqrt{2}$ (d) $50/2\sqrt{3}$

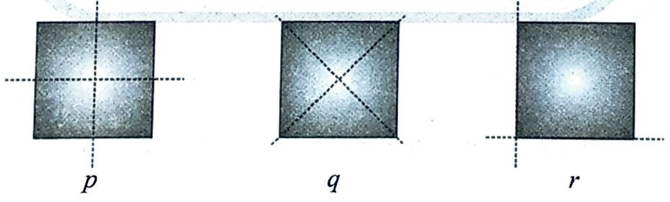
9. We have two spheres, one of which is hollow and the other solid. They have identical masses and moment of inertia about their respective diameters. The ratio of their radius is given by

(a) 5 : 7 (b) 3 : 5 (c) $\sqrt{3}:\sqrt{5}$ (d) $\sqrt{3}:\sqrt{7}$

10. A thin circular disc of mass M and radius R has moment of inertia $\frac{3}{4}MR^2$ about one of chord. Perpendicular distance of chord from centre of disc is

(a) $\frac{R}{2}$ (b) $\frac{R}{\sqrt{2}}$ (c) $\frac{R}{4}$ (d) $\frac{R}{2\sqrt{2}}$

11. The figure shows a thin square sheet of metal of uniform density along with possible choices for a set of principal axes (indicated by dashed lines) of the moment of inertia, lying in the plane of the sheet. The correct choice(s) for the principal axes would be



(a) p, q and r (b) p and r (c) p and q (d) p only

12. Moment of inertia of a solid cube about an axis perpendicular to its face and passing through its centre is [Mass of cube = M, side length = a]

(a) $\frac{Ma^2}{12}$ (b) $\frac{Ma^2}{6}$ (c) $\frac{Ma^2}{3}$ (d) $\frac{Ma^2}{4}$

13. A circular disc of radius R and thickness R/6 has moment of inertia I about its perpendicular axis through centre. The disc is melted and recasted into a sphere M.I. of sphere about its axis is

(a) I/2 (b) I/5 (c) I/6 (d) I/7

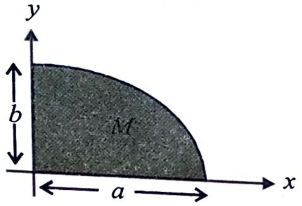
14. Two right circular solid cones have same base radius. Ratio of their heights is 2 : 3. If the two cones have uniform density and their masses are equal, then ratio of moment of inertia about a their axes axis.

(a) 2 : 3 (b) 3 : 2 (c) 9 : 4 (d) 1 : 1

15. A thick hollow sphere of mass 'M' has inner and outer adii R_1 and R_2. Moment of inertia of sphere about its dimeter is

(a) $\frac{1}{5}M\left(R_1^2+R_2^2\right)$ (b) $\frac{2}{3}M\left(R_1^2+R_2^2\right)$ (c) $\frac{1}{3}M\left(R_1^2+R_2^2\right)$ (d) $\frac{2}{5}M\left(\frac{R_2^5-R_1^5}{R_2^3-R_1^3}\right)$

16. A plate shaped as a quadrant of an ellipse is lying in x-y plane as shown in figure. Value of I_{xy} is:

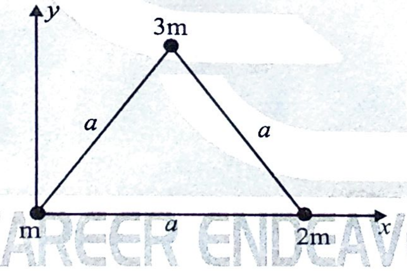


(a) $-\frac{Mab}{2\pi}$ (b) $-\frac{Mab}{\pi}$ (c) $-\frac{Mab}{4\pi}$ (d) $-\frac{Mab}{6\pi}$

17. M.I. of a disc of mass M and radius R is MR^2 about an axis perpendicular to its surface. What is distance of the axis from center of the disc?

(a) $\frac{R}{2}$ (b) $\frac{\sqrt{3}R}{2}$ (c) $\frac{R}{\sqrt{2}}$ (d) $\frac{R}{4}$

18. In the figure shown value of I_{xy} is:



(a) $\frac{-3\sqrt{3}ma^2}{2}$ (b) $\frac{-3\sqrt{3}ma^2}{4}$ (c) 0 (d) $\frac{-\sqrt{3}ma^2}{4}$

19. Four thin rods each of mass M and length l are joined together to form a square loop. M.I. of the loop about its axis is (axis is $\perp$ to plane through centre)

(a) $\frac{ML^2}{3}$ (b) $\frac{ML^2}{4}$ (c) $\frac{4ML^2}{3}$ (d) $\frac{3ML^2}{4}$

20. What is M.I. of the loop in previous question about its diagonal.

(a) $\frac{ML^2}{6}$ (b) $\frac{2}{3}ML^2$ (c) $\frac{ML^2}{3}$ (d) $\frac{ML^2}{4}$

21. We have two spheres, one of which is hollow and the other is solid. The two have equal masses and equal moment of inertia about their diameters, ratio of their radii is (hollow to solid)

(a) 5 : 7 (b) 3 : 5 (c) $\sqrt{3}:\sqrt{5}$ (d) $\sqrt{3}:\sqrt{7}$

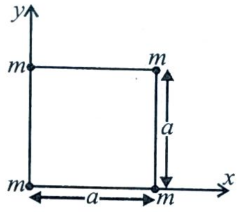
22. 30. For the system of particles shown in the figure which of the following statements is/are correct?

(a) $I_{xy} = -ma^2$

(b) $I_{zz} = 4ma^2$

(c) $\hat{n} = \frac{1}{\sqrt{2}}(-\hat{i} + \hat{j})$ represents a principal axis

(d) x, y axes shown in the figure are not principal axes



23. A solid sphere rolls on a horizontal surface with constant speed. Ratio of rotational kinetic energy and total K.E. is

(a) $\frac{2}{7}$ (b) $\frac{5}{7}$ (c) $\frac{10}{7}$ (d) $\frac{1}{2}$

24. In previous question, ratio of angular momentum of the sphere about a point on its path to its rotational angular momentum is

(a) $\frac{7}{5}$ (b) $\frac{2}{5}$ (c) $\frac{7}{2}$ (d) $\frac{5}{7}$

25. A solid sphere and a hollow sphere roll down an inclined plane. On reaching the bottom of inclined plane ratio of their speed will be

(a) $\sqrt{\frac{25}{14}}$ (b) $\sqrt{\frac{25}{7}}$ (c) $\sqrt{\frac{25}{21}}$ (d) $\sqrt{\frac{5}{7}}$

26. A thin rod of length L is suspended from one end. Time period of oscillation of the rod is

(a) $2\pi\sqrt{\frac{L}{g}}$ (b) $2\pi\sqrt{\frac{L}{2g}}$ (c) $2\pi\sqrt{\frac{2L}{3g}}$ (d) $2\pi\sqrt{\frac{3L}{2g}}$

27. A hollow sphere and a solid cylinder are releasd one by one from same point on an inclined plane. Assume rolling without slipping. Ratio of time taken by cylinder to that by sphere to reach the bottom of inclined plane is

(a) $\sqrt{\frac{10}{9}}$ (b) $\sqrt{\frac{5}{6}}$ (c) $\sqrt{\frac{4}{3}}$ (d) $\sqrt{\frac{3}{5}}$

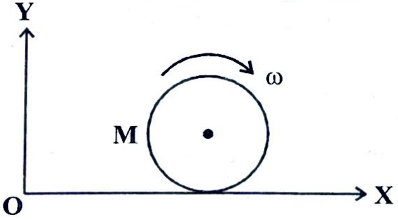
28. A small thin ring of radius 'r' rolls down an inclined plane of inclination θ. Angular acceleration of ring on the inclined plane is

(a) $\frac{g\sin\theta}{r}$ (b) $\frac{g\sin\theta}{2r}$ (c) $\frac{g\sin\theta}{3r}$ (d) $\frac{2g\sin\theta}{r}$

29. A solid cylinder is rolling on a horizontal surface. What fraction is its rotational kinetic energy of total kinetic energy

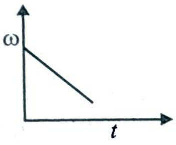
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

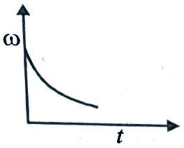
30. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown in figure. The magnitude of angular momentum of the disc about the origin O is

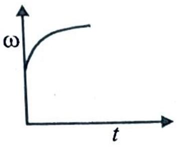


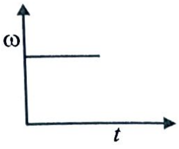
(a) $\frac{1}{2}MR^2\omega$ (b) $MR^2\omega$ (c) $\frac{3}{2}MR^2\omega$ (d) $2MR^2\omega$

31. A spherical ball of ice has radius R_0 and is rotating with an angular speed ω about an axis passing through its centre. At time $t = 0$, it starts acquiring mass because the moisture (at rest) around it starts to freeze on it uniformly. As a result its radius increases as $R(t) = R_0 + \alpha t$, where α is a constant. The curve which best describes its angular speed with time is

(a) 

(b) 

(c) 

(d) 

32. A uniform cylinder of radius r and length ℓ, and a uniform sphere of radius R are released on an inclined plane when their centres of mass are at the same height. If they roll down without slipping, and if the sphere reaches the bottom of the plane with a speed V, then the speed of the cylinder when it reaches the bottom is: [NET]

(a) $V\sqrt{\frac{14r\ell}{15R^2}}$ (b) $4V\sqrt{\frac{r}{15R}}$ (c) $\frac{4V}{\sqrt{15}}$ (d) $V\sqrt{\frac{14}{15}}$

33. A uniform rectangular plate has mass M and sides 'a' and '$2a$'. Moment of inertia of plate about an axis through one corner and perpendicular to plane is

(a) $\frac{5Ma^2}{12}$ (b) $\frac{3Ma^2}{4}$ (c) $\frac{5Ma^2}{3}$ (d) $\frac{7Ma^2}{12}$

Answer Key

1. (a)	**2. (a)**	**3. (d)**	**4. (d)**	**5. (c)**	**6. (d)**	**7. (c)**
8. (a)	**9. (c)**	**10. (b)**	**11. (c)**	**12. (b)**	**13. (b)**	**14. (d)**
15. (d)	**16. (a)**	**17. (c)**	**18. (b)**	**19. (c)**	**20. (b)**	**21. (c)**
22. (a,b,c,d)	**23. (a)**	**24. (c)**	**25. (c)**	**26. (c)**	**27. (a)**	**28. (b)**
29. (b)	**30. (c)**	**31. (b)**	**32. (d)**	**33. (c)**		

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Assignment-9: Classical Mechanics (Central Forces)

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1. In Kepler's problem of planetary motion, the orbits depend on the various values of the eccentricity (e) and energy (E). What values of e and E belong to a hyperbolic orbit?
(a) $E > 0, e = 1$ (b) $E < 0, e = 1$ (c) $E > 0, e = 0$ (d) $E > 0, e > 1$

2. The vector force corresponding to a potential energy $V = kr^3$ is given by
(a) $-3k\,r\,\vec{r}$ (b) $-3k\,r^2\,\vec{r}$ (c) $3k\,r^2\,\vec{r}$ (d) $3k\,r\,\vec{r}$

3. A particle of mass m moves in a central potential $V(r) = Kr^3$ in a circle of radius 'a' its kinetic energy is:
(a) $\frac{3Ka^3}{2}$ (b) $\frac{Ka^3}{2}$ (c) $\frac{3Ka^2}{2}$ (d) $\frac{Ka^2}{2}$

4. Motion of a particle in spherically symmetric attractive potential is:
(a) Always bounded though not in circular orbits
(b) Always in circular orbits and bounded
(c) May be bounded or unbounded depending upon the energy and angular momentum of the particle.
(d) Not bounded at all.

5. Two heavenly bodies of masses 6 and 4 units move under the influence of that mutual central force separating out the centre of mass motion. The resultant one-body motion corresponds to a particle of mass
(a) 10 units (b) 2.4 units (c) 6 units (d) 4 units.

6. Potential energy $V = kr^n$ then the relation between K.E. & P.E. is
(a) $\langle T\rangle = \langle V\rangle$ (b) $\langle T\rangle = \frac{n}{2}\langle V\rangle$ (c) $\langle T\rangle = \frac{3}{2}\langle V\rangle$ (d) $\langle T\rangle = 2\langle V\rangle$

7. If a planet sweeps in equal area in equal time then the following orbit is represented.
(a) Circular orbit (b) Elliptical (c) Hyperbola (d) Parabola.

8. A particle moves under a central force field with the total energy E and the angular momentum $\vec{L}$. For this particle,
(a) E is conserved but $\vec{L}$ is not conserved. (b) E is not conserved but $\vec{L}$ is conserved.
(c) both E and $\vec{L}$ are conserved. (d) neither E nor $\vec{L}$ is conserved.

9. A particle is moving in an inverse square force field. If the total energy of the particle is positive, then trajectory of the particle is:
(a) Circular (b) Elliptical (c) Parabolic (d) Hyperbolic

10. A particle of mass m is located at a distance z along the axis of a uniform disk of mass M and radius R. The gravitational force felt by the mass m is given by

(a) $\frac{2GMm}{R^2}\left(\frac{z}{\left(z^2+R^2\right)^{1/2}}-1\right)$ (b) $\frac{2GMm}{R^2}\left(\frac{z}{\left(z^2-R^2\right)^{1/2}}-1\right)$

(c) $\frac{2GMm}{R^2}\left(\frac{z}{\left(z^2+R^2\right)^{1/2}}+1\right)$ (d) $\frac{2GMm}{R^2}\left(\frac{z}{\left(z^2+R^2\right)^{1/2}}\right)$

11. In a central force field the trajectory of a particle of mass m and angular momentum L in plane polar coordinates is given by $\frac{1}{r}=\frac{m}{L^2}(1+\varepsilon\cos\theta)$ **[GATE 2012]**

Where, ε is the eccentricity of the particle's motion. Which one of the following choices for ε gives rise to a parabolic trajectory ?

(a) $\varepsilon=0$ (b) $\varepsilon=1$ (c) $0<\varepsilon<1$ (d) $\varepsilon>1$

12. A Planet orbits a massive star in a highly elliptical orbit. i.e., the total orbital energy E is close to zero. The initial distance of closest approach is R_0. energy is dissipated through tidal motions until the orbit is circularized with a final radius of R_f. Assume that orbital angular momentum is conserved during the circularization process. Then

(a) $R_f=R_0/2$ (b) $R_f=R_0$ (c) $R_f=\sqrt{2}R_0$ (d) $R_f=2R_0$

13. If, in a Kepler potential, the pericentre distance of a particle in a parabolic orbit is r_p while the radius of the circular orbit with the same angular momentum is r_c, then

(a) $r_c=2r_p$ (b) $r_c=r_p$ (c) $2r_c=r_p$ (d) $r_c=\sqrt{2}\,r_p$

14. How is your weight affected if the Earth suddenly doubles in radius, mass remaining the same ? **[JEST 2015]**

(a) Increases by a factor of 4 (b) Increases by a factor of 2

(c) Decreases by a factor of 4 (d) Decreases by a factor of 2

15.✓ A satellite is moving in a circular orbit around the Earth. If T, V and E are its average kinetic, average potential and total energies, respectively, then which one of the following options is correct ? **[GATE 2015]**

(a) $V=-2T;\ E=-T$ (b) $V=-T;\ E=0$

(c) $V=\frac{-T}{2};\ E=\frac{T}{2}$ (d) $V=\frac{-3T}{2};\ E=\frac{-T}{2}$

16. A satellite moves around the earth in a circular orbit of radius R centered at the earth. A second satellite moves in an elliptic orbit of major axis 8R, with the earth at one of the foci. If the former takes 1 day to complete a revolution, the latter would take

(a) 21.6 days (b) 8 days (c) 3 hours (d) 1.1 hour

17. An artificial satellite revolves around the earth in circular orbit, time of revolution of satellite is T and its energy is E. If the satellite goes into other circular orbit in which its time of revolution becomes 8T then its energy in new orbit will be comes E'. Then value of E/E' is ________

18. The greatest and least velocities of a planet around the Sun are v_1 and v_2. Eccentricity of its orbit is

(a) $\frac{v_1 - v_2}{v_1 + v_2}$ (b) $\frac{v_1^2 - v_2^2}{v_1^2 + v_2^2}$ (c) $\frac{\sqrt{v_1} - \sqrt{v_2}}{\sqrt{v_1} + \sqrt{v_2}}$ (d) $\frac{\sqrt{v_1}}{\sqrt{v_2}}$

19. In the previous question, if semi major axis of the planet is 'a' then time period for one revolution around the sun is

(a) $\frac{2\pi a}{(v_1 + v_2)}$ (b) $\frac{2\pi a}{\sqrt{(v_1 v_2)}}$ (c) $\frac{\pi a}{(v_1 + v_2)}$ (d) $\frac{\pi a}{\sqrt{(v_1 v_2)}}$

20. For a particle moving under central force, which one of the following statement is always true?

(a) $\dot{r} = 0$ (b) $\ddot{\theta} = 0$ (c) $2\dot{r}\dot{\theta} + r\ddot{\theta} = 0$ (d) $\ddot{r} - r\dot{\theta}^2 = 0$

21. For a particle moving under a central force which one of the following quantities is not constant.

(a) Total mechanical energy (b) Momentum
(c) Angular momentum (d) Areal velocity

22. In central force motion a particle of mass 'm' has angular momentum 'L'. Areal velocity of the particle is

(a) $\frac{L}{m}$ (b) $\frac{2L}{m}$ (c) $\frac{L}{2m}$ (d) $\frac{4L}{m}$

23. For $F = \frac{k}{r^n}$ equation of orbit of a particle is $r = a(1 + \cos\theta)$. What is value of n?

(a) 2 (b) 3 (c) 4 (d) 5

24. A particle of mass m is moving under the central potential $V(r) = -\frac{k}{r}$. If energy of particle is zero and its angular momentum is L then its distance of nearest approach from centre of force is

(a) $\frac{L^2}{mk}$ (b) $\frac{L^2}{2mk}$ (c) $\frac{2L^2}{mk}$ (d) $\frac{L^2}{mk^2}$

25. A particle moving under central force has mass m, energy E and angular momentum L. If $r = r_0$ be turning point then speed of the particle at this point is

(a) $\sqrt{\frac{2E}{m}}$ (b) $\frac{L}{mr_0}$ (c) $\sqrt{\frac{2}{m}\left(E - \frac{L^2}{mr_0^2}\right)}$ (d) $\sqrt{\frac{2}{m}\left(E + \frac{L^2}{mr_0^2}\right)}$

26. A particle of mass m moving in circular orbit under a central force $f = -Kr^3$, has angular momentum L. If radius of circular orbit is doubled then its angular momentum will become

(a) 2L (b) 4L (c) 8L (d) $2^{3/2}$ L

Answer Key

1. (d)	**2. (a)**	**3. (a)**	**4. (c)**	**5. (b)**	**6. (b)**	**7. (b)**
8. (c)	**9. (d)**	**10. (a)**	**11. (b)**	**12. (d)**	**13. (a)**	**14. (c)**
15. (a)	**16. (b)**	**17. (4)**	**18. (a)**	**19. (b)**	**20. (c)**	**21. (b)**
22. (c)	**23. (c)**	**24. (b)**	**25. (b)**	**27. (c)**		

CAREER ENDEAVOUR

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CSIR-UGC-NET/JRF | GATE PHYSICS

Assignment-10: Classical Mechanics (Non-Inertial Frame of Reference)

1. A girl measures the period of simple pendulum inside a stationary lift and finds it to be T seconds. If the lift accelerates upward with an acceleration g/4, then the period of the pendulum will be

(a) T (b) T/4 (c) $2T/\sqrt{5}$ (d) $2T\sqrt{5}$

2. The period of a simple pendulum inside a stationary lift is T. If the lift accelerates downwards with an acceleration $g/4$, the period of the pendulum will be

(a) T (b) $T/4$ (c) $2T/\sqrt{3}$ (d) $2T/\sqrt{5}$

3. The motion of a particle of mass m is described in a non-inertial frame of reference that is rotating with a uniform angular velocity ω. If r denotes the position of the particle in the non-inertial frame

(a) The centrifugal force on the particle is $-m\omega^2\dot{r}$

(b) The centrifugal force on the particle is $-m\vec{\omega}\times(\vec{\omega}\times\vec{r})$

(c) The Coriolis force on the particle is $-m\left(\vec{\omega}\times\frac{d\vec{r}}{dt}\right)$

(d) The Coriolis force on the particle $-2m\omega^2 r$

4. A horizontal circular platform rotates with a constant angular velocity Ω directed vertically upwards. A person seated at the centre shoots a bullet of mass 'm' horizontally with speed 'v'. The acceleration of the bullet, in the reference frame of the shooter, is **[NET June 2012]**

[✓ (a) $2v\Omega$ to his right] (b) $2v\Omega$ to his left

(c) $v\Omega$ to his right (d) $v\Omega$ to his left

5. A person standing on a rotating disc sees that everything around him is rotating which of the following statements given by the person is correct.

(a) Both coriolis and centrifugal forces on all objects are radially outward

(b) Both coriolis and centrifugal forces on all objects are radially inward

(c) On all objects coriolis force is radially inward and centrifugal force is radially outward

(d) Centrifugal force and coriolis force are equal in magnitude but opposite direction.

6. A stone is dropped from a height 100m at equator. What is east-ward deviation of the particle (approximately)

(a) 3.15 cm (b) 2.17 cm (c) 1.08 cm (d) 2.73 cm

7. In the previous question if the location is at latitude 60° in northern hemisphere then eastward deviation will be

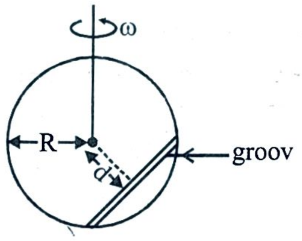
(a) 3.15 cm (b) 2.17 cm (c) 1.08 cm (d) 2.73 cm

8. If ω be angular velocity of the earth's spinning motion and R be the radius of the earth, with what speed should a train move along equator so that magnitude of centrifugal and coriolis forces and acting on the train become equal.

(a) ωR (b) $\frac{\omega R}{2}$ (c) $2\omega R$ (d) $\frac{2\omega R}{3}$

9. Particle of mass 2kg is moving in north east direction on earth's surface at a location where latitude is 45° N. Find magnitude of coriolis force experience by the particle if its speed is 200 m/s.
(a) 0.05 N (b) 0.5 N (c) 5 N (d) 0.25 N

10. A circular disc is rotating about its axis with uniform angular velocity ω. There is a groove along a chord of the disc. An insect starts crawling along the groove. If the insect starts at $t = 0$ from the middle point, and moves with constant speed v_0 w.r.t. the disc. What is centrifugal force on the insect at time t.



(a) $m\omega^2 d$ (b) $m\omega^2 v_0 t$ (c) $m\omega^2(d+v_0 t)$ (d) $m\omega^2\sqrt{d^2+v_0^2 t^2}$

11. An observer is located on a horizontal, circular turntable which rotates about a vertical axis passing through its centre, with a uniform angular speed of 2 rad/sec. A mass of 10 grams is sliding without friction on the turntable. At an instant when the mass is at a distance of 8 cm from the axis, it is observed to move towards the center with a speed of 6 cm/sec. The net force on the mass, as seen by the observer at that instant, is
(a) 0.0024N (b) 0.0032 N (c) 0.004 N (d) 0.006 N

12. A very long rod rotates about a pivot with a constant angular velocity ω. A bead is constrained to slide along the rod without friction. At time t = 0, the bead is at rest a distance d away from the pivot. Its distance r(t) from the pivot at time t is
(a) $d\sinh(\omega t)$ (b) $d\sin(\omega t)$ (c) $d\cosh(\omega t)$ (d) $d\cos(\omega t)$

13. Which of the following classic experiments provides unambiguous proof that the Earth is a non-inertial frame of reference with respect to the fixed stars? **[TIFR 2013]**
(a) Fizeau's rotating wheel experiment (b) Foucault's pendulum experiment
(c) Newton's coin-and-feather experiment (d) Michelson-Morley experiment

Answer Key

1. (c) **2. (c)** **3. (b)** **4. (a)** **5. (c)** **6. (b)** **7. (c)**

8. (b) **9. (a)** **10. (d)** **11. (c)** **12. (c)** **13. (b)**

CSIR-UGC-NET/JRF | GATE PHYSICS

Assignment-11 : Classical Mechanics (Collision)

1. Two particles approach each other with different velocities. After collision, one of them is found to have momentum $\vec{p}$ in their centre of mass frame. In the same reference frame, the other particle must have momentum

(a) Zero (b) $\frac{-\vec{p}}{2}$ [✓ (c) $-\vec{p}$] (d) $-2\vec{p}$

2. A particle of mass m collides elastically with another particle of mass 2m initially at rest. if speed of first particle in center of mass frame after collision is V_0. Then speed of first particle in lab frame before collision is:

(a) $\frac{3V_0}{2}$ (b) $\frac{V_0}{2}$ (c) $\frac{2V_0}{3}$ (d) $2V_0$

3. After a perfectly elastic collision of two identical balls, one of which was initially at rest, the velocities of both the balls are non-zero. The angle θ between the final velocities (in the lab frame) is **[NET Dec. 2016]**

(a) $\theta = \frac{\pi}{2}$ (b) $\theta = \pi$ (c) $0 < \theta < \frac{\pi}{2}$ (d) $\frac{\pi}{2} < \theta \leq \pi$

4. A particle of mass m collides elastically with another particle of mass $\sqrt{3}m$ which is at rest. If in lab frame first particle scatters at angle 60° to initial direction of motion. Then in center of mass frame angle of scattering will be

(a) 60° (b) 15° (c) 90° (d) 45°

5. Two particles of masses *m* and 2*m* are moving in same direction with velocities $2v_0$ and v_0 respective. If they collide elastically then speed of 2*m* after collision in centre of mass frame will be

(a) v_0 (b) $2v_0$ (c) $\frac{2v_0}{3}$ (d) $\frac{v_0}{3}$

Answer Key

1. (c) **2. (a)** **3. (a)** **4. (c)** **5. (d)**